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ABSTRACT

This study evaluates materials developed for low achievers by secondary mathematics teachers who participated in a summer conference. Fifteen pairs of experimental/control classes were involved. Materials were evaluated by data from achievement tests and attitude instruments. In addition, the teachers using the materials were asked for suggestions to improve the materials. School visitations to obtain teacher, student, and administrator reactions were made. Findings and conclusions based on the findings are reported. Students and teachers reported that most of the materials were used to supplement regular instruction. Comments indicated that students and teachers enjoyed using the materials more than regular materials. Test data indicated no significant increase in achievement or attitudes by those using the materials. (MK)

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AN EVALUATION OF ACTIVITY ORIENTED MATERIALS DEVELOPED TO HELP THE LOW ACHIEVER ATTAIN BASIC MATHEMATICAL COMPETENCIES

by

Jackie L. Beal

A DISSERTATION

Presented to the Faculty of

The Graduate College in the University of Nebraska

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For the Degree of Doctor of Philosophy

Department of Secondary Education

Under the Supervision of Professor Milton W. Beckmann

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J.L.B.

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CHAPTER I

INTRODUCTION

BACKGROUND FOR STUDY

Since approximately 1955, we have expended much time and effort in providing for the high achiever in mathematics through specially designed programs as well as special training for the teacher. Only recently, since the Elementary and Secondary Education Act of 1965, has the emphasis shifted to the educationally disadvantaged.

In order to help meet the needs of the disadvantaged student, Dr. Milton W. Beckmann applied for and received National Science Foundation funds for a three-week summer conference to be held in the summer of 1971. The purpose of this conference was to:

1. better prepare qualified secondary teachers to teach mathematics to low achievers; and
2. collect, review and develop new methodologies and materials for teaching the low achiever.

Twenty-three secondary teachers from the United States and one from American Samoa participated in the conference. They represented schools from many different socio-economic classes as well as geographic regions. For example, one participant was an administrator from the Chicago Public Schools who was in a position to influence over 577,000 students while another, a classroom teacher from

Culbertson, Nebraska, teaches approximately 50 students. One participant worked primarily with Indian students in New Mexico, while another worked in a suburban area of St. Louis, Missouri.

One of the goals of this conference was to develop materials to be used in teaching the academically disadvantaged student. Realizing the classroom teacher has a limited amount of time to be used in collecting and reviewing instructional materials, it was decided to organize the materials around specific mathematical competencies needed by all citizens and therefore of interest to all teachers of low achievers.

The National Council of Teachers of Mathematics was selected to be the source for these competencies. In 1945, the Commission on Post War Plans of the National Council of Teachers of Mathematics, published in Volume XXXVIII of The Mathematics Teacher, a list of 29 questions that were to form the basis for assessing one's basic mathematical competencies. These 29 questions have been the basis for deciding what should be the mathematical competencies of all citizens since their publication in 1945. However, the National Council of Teachers of Mathematics recognized the advances in technology made since the issuance of this report and established the Committee on Basic Mathematical Competencies to re-define what might be considered the competencies of all enlightened citizenship in contemporary society. This task has been completed and the committee will publish guidelines in the fall of 1972. A preliminary draft of

this report was obtained for the 1971 Conference on the Low Achiever and was used as a source for selecting the competencies for which materials would be developed. A complete listing of the competencies is found in Table I. Not only would the materials be competency based, but the materials would be "keyed" to specific behavioral objectives that would reflect the attainment of these competencies.

TABLE I

MINIMUM BASIC MATHEMATICAL COMPETENCIES AND SKILLS NEEDED BY ENLIGHTENED CITIZENS

Mathematical Competencies Necessary for Enlightened Citizenship in Contemporary Society
The mathematically competent citizen of the contemporary society
A. Understands the nature of mathematics. This implies the following:
1. Understanding the deductive nature of mathematics by recognizing that mathematics consists of undefined terms, definitions, axioms and theorems.
2. Ability to prove theorems in some mathematical context.
3. Ability to recognize a valid argument and an invalid one.
4. Knowledge of properties of operations, for example commutative and associative properties of addition of real numbers.
5. Familiarity with some mathematical systems, for example a field and a group.
6. Recognition that various concepts and operations are related to each other; for example, that subtraction and addition are inverse operations.

TABLE I (continued)

7. Ability to perceive patterns displayed by means of sequences of specific instances.

B. Understands the role of mathematics in society. This implies the following:

1. Knowing the ways in which computers are used in sciences, technology, business and government.

2. Recognizing the potential as well as the limitations of electronic computers.

3. Recognizing the historical milestones in the development of mathematical ideas which served man in solving his problems and which aided him in the control of his environment.

4. Being aware of the great frequency with which mathematical skills are used by individuals in their daily lives.

5. Being able to make use of mathematics as a tool in solving personal problems.

6. Recognizing that there are problems which, by their nature, do not lend themselves to solution by mathematical methods.

7. Recognizing that some professions require knowledge of the most sophisticated and complex mathematical techniques.

8. Recognizing that lack of certain mathematical knowledge on the part of an individual may preclude him from making a contribution to society which is commensurate with his capabilities.

9. Being aware of the fact that mathematics finds direct applications not only in the natural sciences but in the behavioral and social sciences and arts as well.

C. Can use mathematics as a tool in performance of his duties.

1. Numbers and Numerals

a. Express a rational number using decimal notation.

TABLE I (continued)

b. List the first ten multiples of 2 through 12.
c. Use both positive and negative numbers in problem solving.
d. Identify and use in problem solving the least common multiple and greatest common divisor of two numbers.
e. Recognize the digit, its place value, and the number represented through billions.
f. Describe a given positive rational number using decimal, percent, or fractional notation.
g. Convert Roman Numerals from decimal numerals and conversely.
h. Identify significant figures in a given number.
i. Represent large and small numbers using scientific notation.

2. Operations and Properties

a. Write an equivalent faction for given fractions, such as 1/2, 2/3, and 3/5.
b. Use the standard algorithms for the operations of arithmetic of whole rational numbers.
c. Estimate product and quotients of two numbers through thousands.
d. Recognize and use the 1 and 0, order, and grouping properties of addition and multiplication.
e. Solve addition, subtraction, multiplication and division problems with fractions having denominators less than twenty.
f. Solve percent problems using addition, subtraction, multiplication and division.
g. Describe the basic properties of equality, the relationship of addition to subtraction and of multiplication to division, and the distributive property of multiplication over addition.
h. Add, subtract, multiply, and divide denominate numbers.

3. Mathematical Sentences

a. Construct a true mathematical statement from a given verbal problem.
b. Solve simple linear equations like a+3=12, 16-n=4, n/3=7, and 4a-2=18.

TABLE I (continued)

c. Translate mathematical sentences into verbal problems.
d. Translate a graphical representation of two variables into a mathematical sentence.

4. Geometry

a. Describe parallel lines, perpendicular lines and intersecting lines using drawings or intuitive concepts.
b. Classify simple plane figures by distinguishing some of their properties.
c. Compute the perimeter of a given polygon.
d. Compute the area of a rectangle and of a triangle.
e. Construct bisectors of lines and angles with a compass and straight edge.
f. Identify the conditions for similarity of triangles and use the properties of similarity to solve problems.

5. Measurement

a. Apply the common English measures of length, volume (dry or liquid), weight, time, money, and temperature.
b. Convert from one measure to an equivalent one with larger or smaller units in the English System.
c. Use metric units of length, mass, and volume in making measure.
d. Recognize that no measurement is precise.
e. Convert, using tables, English to metric measures and conversely.
f. Use standard measuring devices of length, area, volume, time, and temperature to make measurements.
g. Round off measurements to the nearest given unit of the measuring device used such as rulers, protractors and thermometers.

6. Relations and Functions

a. Use percent as a special form of ratio to describe a relationship of two quantities.
b. Interpret information from a graphical representation of a function.
c. Apply the concepts of ratio and proportion to construct scale drawings, and to determine percent and other relations.

TABLE I (continued)

d. Write simple sentences showing the relations =, <, > for two given numbers.

7. Probability and Statistics

a. Predict the probability of simple events occurring.
b. Distinguish measures of central tendency from given numerical data.
c. Analyze and solve simple probability problems such as tossing coins or drawing one red marble from a set containing one red and four white marbles.
d. Estimate answers of computational problems.

8. Graphing

a. Determine measures of the real object from a scale drawing.
b. Construct a scale drawing of simple objects.
c. Construct a graph indicating the relationship of two variables from a given set of data.
d. Interpret information from a bar graph, circle graph, line graph and tables.

9. Mathematical Proof

a. Seek counter examples to determine validity of a statement.
b. Detect and describe flaws and fallacies in advertising and propaganda where statistical data and inferences are employed.
c. Gather and present data to support an inference or argument.
d. Construct proofs from patterns involving numerical data.

10. Business and Consumer Mathematics

a. Maintain personal bank records.
b. Plan a budget including record keeping of personal and travel expenses.
c. Apply simple interest formulas to a problem in time payment buying.
d. Use adders and/or calculators to solve addition, subtraction, multiplication, and division problems.

TABLE I (continued)

e. Compute taxes, investment return, and capital gains.
f. Estimate the real cost of an article by taking into account depreciation, expenses, salvage value and financing costs.
g. Read maps and estimate distance between points on the map.

11. Algebra and Trigonometry

a. Describe verbal problems using common algebraic symbolism.
b. Solve problems using the sine, cosine and tangents from tables.
c. Apply the pythagoreon theorem to solve right triangle problems.
d. Manipulate simple algebraic expressions of rational numbers and inequalities.

The conference materials were also designed to stress the student becoming an active participant in the learning process. Support for active participation can be found in Goals for School Mathematics, where it is stated that "Psychologically speaking, mathematics is something which people do, it is not something that they receive in a passive sense."[1] Also according to E. R. Hilgard, "Active participation by a learner is preferable to passive reception when learning, for example, from a lecture or motion picture."[2]

[1] Goals for School Mathematics, The Report of the Cambridge Conference on School Mathematics (Boston, Mass.: Houghton Mifflin Company), p. 28.

[2] E. R. Hilgard, Theories of Learning (2nd ed.; New York: Appleton-Century-Crofts, Inc., 1956), p. 486.

Herbert Fremont, author of a book dealing with the teaching of secondary school mathematics states, "Introduce any topic in a way that makes the student an active rather than a passive participant."[3]

Recognizing the need for active participation, we have long used games, puzzles, and activities in the classroom. However, these have normally been used as motivational devices. This conference developed units of instruction in which activities were selected so that they not only provided motivation but also helped the student acquire certain basic mathematical competencies. Since teachers are limited in their material resources, the units were constructed so that the teacher may use one or more of the activities in helping the students achieve specific behaviorally stated objectives. Also the units were constructed with the thought that they would supplement rather than replace the regular classroom materials and would be used only with those students who showed deficiencies after working through the regular materials. It was the feeling of the conference members that if the materials were going to be used then they must make the teacher's task easier.

A concern of both the author and the director was that many materials are written without any evaluation being made to determine their effectiveness. Therefore, the following study was proposed as a method for providing an evaluation of the effectiveness of the

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[3]Herbert Fremont, How to Teach Mathematics in Secondary Schools (Philadelphia: W. B. Saunders Company, 1969), p. 78.

conference materials in meeting the needs of the low achiever.

STATEMENT OF THE PROBLEM

The first purpose of this study was to evaluate the effectiveness of the materials developed in the National Science Foundation Conference, based on active involvement in the learning process by the learner, in helping the low achiever attain basic mathematical competencies. The second purpose of this study was to determine if the low achiever has a more positive attitude towards mathematics after using the materials.

DEFINITION OF TERMS

Low achiever. Designated by the individual schools. However, in most cases a student who scored below the 30th percentile on a standardized achievement test was selected.

Unit. Written materials consisting of the following:

1. a mathematics competency
2. instructional objectives
3. one or more activities each of which consists of:
   a. suggested materials
   b. instructions to the student
   c. suggested teaching strategies

Regular materials. Materials that the teacher would normally use in teaching the class. In many cases, this would be a textbook.

Achievement. For the purpose of this study, achievement was determined by the score attained on a mathematics achievement test constructed by the author.

Attitude toward mathematics. For the purpose of this study, attitude toward mathematics was determined by the scores attained on The Revised Math Attitude Scale.[4]

Experimental class. A group of students assigned to use part or all of the materials prepared by the summer conference to supplement or replace the regular materials.

Control class. A group of students assigned to use the same materials as the experimental class except for the conference materials.

HYPOTHESES TO BE TESTED

1. There are no significant differences between the attitude toward mathematics of the experimental classes and the attitude toward mathematics of the control classes.

2. There are no significant changes in attitude toward mathematics by the experimental classes.

3. There are no significant changes in attitude toward mathematics by the control classes.

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[4]Lewis R. Aiken, The Revised Mathematics Attitude Scale (Athens, Georgia: University of Georgia).

4. There are no significant differences between the achievement of the experimental classes and the achievement of the control classes.

SCOPE AND DELIMITATION

The investigation is basically concerned with providing an objective evaluation of an activity approach to the teaching of basic mathematical competencies to the low achiever in mathematics. Indirectly, it will provide an evaluation of one of the outcomes of the Summer Conference on the Low Achiever in Mathematics.

This study will not determine which activity or combination of activities will be most effective in helping the low achiever overcome a deficiency in mathematics. Also, it will not provide an evaluation of each unit, but rather will provide an evaluation of various combinations of units.

The extent to which the study can be generalized is limited by the number of schools involved as listed below. However, it is hoped that by using the schools in six different states, this deficiency may be minimized.

1. Total number of teachers: thirteen
2. Total number of classes: thirty
3. Total number of experimental classes: fifteen
4. Total number of control classes: fifteen
5. Total number of students: 517
6. Total number of schools: ten

7. Location and types of schools:

   Freeport, Illinois, Middle School, Public

   Park Ridge, Illinois, Junior High, Public

   Indianapolis, Indiana, Junior High, Public

   Omaha, Nebraska, Senior High, Public

   Tilden, Nebraska, Senior High, Public

   New Hartford, New York, Junior High, Public

   Cincinatti, Ohio, Senior High, Parochial

   Cleveland, Ohio, Junior High, Public

   Columbus, Ohio, Senior High, Parochial

   Appomatax, Virginia, Senior High, Private

8. Duration of study: September, 1971 through January, 1972

OVERALL PLAN

Ten teachers who participated in the conference plus three teachers who did not participate in the conference agreed to use parts or all of the conference materials during the fall semester of the 1971-1972 school year. Each teacher would teach at least two classes, one experimental and one control. Students would either be assigned at random or by computer to the experimental and control classes. The teachers were allowed to select units to be used in the experimental classes as long as they used at least six units. Also

they were asked to indicate what activities took place in the control classes during the time the experimental classes were using conference materials.

The Revised Math Attitude Scale, developed by Lewis R. Aiken at the University of Georgia, was administered as a pretest and a posttest to all students who participated in the study. Also the writer constructed a fifty-two item achievement test, called Beal's Mathematics Competency Test, to measure the twenty-six competencies for which the conference materials were written. This test was administered to participating students as a posttest in January of 1972. A computer program was written to allow the writer to select from the fifty-two items on the achievement test only those pertaining to the units used by individual teachers in their experimental classes. In addition to the above forms of written evaluation, a visit was made to six of the ten participating schools where teachers, administrators, and students were interviewed to determine their reaction to the conference materials.

CHAPTER II

REVIEW OF RELATED LITERATURE

Since the enactment of the Elementary and Secondary Education Act of 1965, there has been a noticeable increase in interest for the low achiever in mathematics. This is evident by the many requests for workshops, institutes, and conferences dealing with the low achiever. As recently as the spring of 1972, Dr. Milton W. Beckmann received approximately 750 requests for applications for a three-week summer conference concerned with problems of teaching the low achiever. This conference could only accommodate twenty-five participants.

Concern for the low achiever is best summarized in the following statement concerning individual differences.

Smith wrote:

> Plato said that every individual should perform those tasks for which he was best qualified by nature. Aristotle recognized individual differences and the influence of education upon them. The Romans and the educators of the Renaissance also saw that not every one has outstanding gifts or outstanding ability. Rousseau said that each child has his own cast of mind in accordance with which he must be directed. But Francis Galton was the first, about one hundred years ago, to undertake a systematic and statistical study of individual differences. Extensive psychological study of the more complex mental processes began with the Binet-Simon intelligence scale of 1905.[1]

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[1]Rolland R. Smith, "Providing for Individual Differences," The Learning of Mathematics: Its Theory and Practice, Twenty-first Yearbook of the National Council of Teachers of Mathematics (Washington, D.C.: The Council, 1953), p. 271.

If a true commitment to individualized instruction is accepted, one cannot escape our responsibility to provide special programs for the low achiever. The review of literature will be divided into six main topics: who are the low achievers; characteristics of the low achiever; why a special program for the low achiever; what mathematics for the low achiever; research relative to instructional methods to be used with the low achiever; and research relative to materials to be used by the low achiever.

WHO ARE THE LOW ACHIEVERS?

John Ogle has defined the low achiever as "that group of students functioning below grade level but not low enough to need special education."[3] Lauren G. Woodby,[3] when reporting on the conference for the Low Achiever in Mathematics held in Washington, D.C., indicated that we are considering the lowest 30 per cent based upon achievement in school mathematics. The low achiever has also been referred to as below average achievers, underachievers, culturally deprived, educationally deprived, culturally disadvantaged, reluctant learners, slow learners, rejected learners, underprivileged, lower

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[2]John W. Ogle, "Unfinished Revolution: Mathematics for Low Achievers," High School Journal, LIII (February, 1970), 298.

[3]Lauren G. Woodby, The Low Achiever in Mathematics, OE 2906, Bulletin 1965, No. 31 (Report of a conference held in Washington, D.C., March 25-27, 1964, sponsored jointly by the U.S. Office of Education and the National Council of Teachers of Mathematics).

ability students, students of limited interest or ability, mathematically underdeveloped children, and the forgotten people of education.[4] Probably the two terms used most often to describe the type of student under consideration are low achiever and underachiever. An underachiever, according to John Snellgrove, is a student who has "a 1.25 standard deviation difference between grade point average in major subjects and Primary Mental Abilities Test score, the latter being the higher of the two."[5] Ross[6] defined underachievers as students who have average to above average intelligence and who were two years or more below their mental grade levels in arithmetic achievement as measured by a standardized arithmetic test. T. Jones,[7] writing in The Mathematics Teacher, reported of dividing a general mathematics class into two groups, low achievers with IQ scores less than 85 and underachievers with IQ scores greater than 85.

CHARACTERISTICS OF THE LOW ACHIEVER

Some studies have been conducted to identify the characteristics

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[4] Ogle, loc. cit.

[5] John Louis Snellgrove, "A Study of Relationships Between Certain Personal and Socio-Economic Factors and Underachievement" (University of Alabama, 1960), Dissertation Abstract, 21:1859, January 1961.

[6] Roman Royal Ross, "A Case Study Description of Underachievers in Arithmetic" (University of Oregon, 1962), Dissertation Abstract, 22:2294, January, 1962.

[7] T. Jones, "Effect of Modified Programmed Lecture and Mathematical Games Upon Achievement and Attitude of Ninth-Grade Low Achievers in Mathematics," The Mathematics Teacher, LXI (October, 1968), 630-37.

of the low achiever. These studies have helped to better describe who is the low achiever by identifying his characteristics.

Jacobs reported on a study to determine the achievement level of 293 teen-age, slow learners in special classes in Cincinnati Public Schools. The students ranged in age from 13 through 18 with a median age of 15 years 6 months. The following conclusions were drawn from an analysis of test scores:

1. The average total grade equivalent level of the slow learners studied was 4.0 while the achievement in reading, arithmetic, and language was 3.8, 4.3, and 3.7, respectively.
2. Girls were significantly higher than boys in all subtests except arithmetic where there was no significant difference.
3. For both boys and girls, the arithmetic subtest grade equivalent proved significantly higher than either reading or language scores. The latter two showed no significant difference.
4. The mean grade equivalent on the arithmetic fundamentals section of the arithmetic test was 4.6 while reasoning was 4.0.[8]

The author would like to emphasize that these students ranged in age

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[8]James N. Jacobs, "A Study of Performance of Slow Learners in Cincinnati Public Schools on Mental and Achievement Tests," American Journal of Mentally Defective, LXII (September, 1957), 238-43.

from 13 through 18.

R. I. Hoffman,[9] Professor of Mathematics at the University of Denver, listed the following characteristics of the low achiever:

1. A record of failure in mathematics.
2. Achievement scores three or four years below grade level in mathematics.
3. Reading and comprehension difficulties in many cases.
4. Quick conclusions formed without due consideration of facts.

John W. Ogle[10] listed the following characteristics of the low achiever: history of failure, poor self-image, poor reading ability, little interest in mathematics, high rate of absenteeism, poor memory, short attention span, emotional and social immaturity, difficulty with abstractions, and failure to see the practical use of mathematics as it applies to him.

John Snellgrove[11] conducted a study to determine what relationships, if any, existed between personal and socio-economic factors and underachievement of high school students as junior and senior high school. After administering the California Test of Personality,

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[9] R. I. Hoffman, "Slow Learner; Changing His View of Mathematics," Bulletin of the National Association of Secondary School Principals, LII (April, 1968), 86-97.

[10] Ogle, loc. cit.

[11] Snellgrove, loc. cit.

the Fields of Study Motivation Record, and a questionnaire to 213 underachievers, he arrived at the following conclusions:

1. There is a larger number of male than female underachievers.
2. There is a higher number of underachievers at the 7-9 level than at the 10-12 level.
3. There is a positive correlation between underachievement and motivation in mathematics.
4. Underachievers scored below CTP test norms on personal, social, and total adjustment.
5. Personality maladjustments decreased in underachievers as a group from 7 through 12 grades.
6. Underachievers do not identify more with other underachievers than they do with individuals outside this group.
7. Setting up occupational goals was not related to underachievement.

A study to determine differences in interest and attitude between underachievers in mathematics and a control group was conducted by Mary Ann Beaton[12] during the 1964-65 school year. Percentile scores on the computational scale and the literacy scale of

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[12]Mary Anne Beaton, "A Study of Underachievers in Mathematics at the Tenth Grade Level in Three Calgary High Schools" (Northwestern University, 1966), Dissertation Abstract, 27A:3215-3216, April, 1967.

the Kuder-Richardson Preference Record-Vocational were obtained for both experimental and control classes. Also, the Storey and Guhaty Attitude Scale, a questionnaire of ten items, was used to determine attitude of students toward mathematics. Finally, an interview form for parents was used to determine attitudes of parents toward mathematics. Upon examining the results of the data obtained with the above instruments, the investigator arrived at the following conclusions:

1. Mean score for underachievers in mathematics on the computational interest scale was significantly lower than control students.
2. Mean scores for underachievers on the literacy scale were significantly higher than for control students.
3. Underachievers showed a less favorable attitude toward mathematics.
4. Parents of experimental students had mean scores significantly less than control students' parents on attitude toward mathematics.
5. Parents of control students expressed the opinion that knowledge of mathematics through twelfth grade was important.

The difficulty in identifying the low achiever in mathematics is graphically pointed up in a study which was

conducted by Fenner[13] in 1965. His study was concerned primarily with (1) a definition of the underachiever, (2) age of onset of underachievement, (3) chronicity of underachievement, and (4) whether underachievers selected on basis of grade point averages in primary grades would perform at a higher level on an achievement measure in high school than would subjects selected in the same early grades when Stanford Achievement Reading or Arithmetic tests were the criteria. He collected data on 42 boys and 42 girls who were juniors and seniors in a suburban high school and arrived at the following conclusions:

1. There was little overlap of subjects selected as underachievers by the three criteria.

2. Few subjects, who were not selected by tenth grade, began to more consistently underachieve.

3. Consistent identification of an underachiever was non-existent.

He concluded that relative level of achievement probably is quite stable, but current measures and methods of studying underachievement do not permit accurate or reliable observation of it.

Finally, if we consider the low achievers' computational level and reading level, we find support for the argument that they do not possess basic computational skills and may be hampered by

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[13]Elmer David Fenner, Jr., "An Investigation of the Concept of Underachievement" (Western Reserve University, 1965), Dissertation Abstract, 27A:600, September, 1966.

poor reading skills. Ross[14] administered a battery of tests and conducted interviews to measure various dimensions of human behavior among twenty sixth- and seventh-grade arithmetic underachievers of average or above average intellectual ability. The results he obtained are as follows:

1. Mean percent accuracy in the basic computational processes were as follows: addition, 88 percent; subtraction, 79 percent; multiplication, 66 percent; division, 33 percent; addition of fractions, 18 percent; subtraction of fractions, 14 percent; multiplication of fractions, 8 percent; division of fractions, 2 percent.

2. Satisfactory reasoning in word problems involving addition and subtraction of whole numbers, but frequent errors in multiplication and division of whole numbers as well as all processes involving common fractions.

3. Sixteen of the twenty subjects were one or more years below their mental age in reading ability.

4. Subjects were characteristically withdrawn and defeated in their attitudes toward school and society.

5. School records indicated subjects were underachievers generally in school subjects other than arithmetic.

Theodora Nelson,[15] in 1956, reported the results of a general mathematics test given for the previous five years as part of the Inter-High School Contests given at Kearney State College. Her findings were consistent with those of Ross. She reported weaknesses

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[14]Ramon Royal Ross, "A Case Study Description of Underachievers in Arithmetic" (University of Oregon, 1962), Dissertation Abstract, 22:2294, January, 1962.

[15]Theodora Nelson, "Results of General Mathematics Test," Arithmetic Teacher, III (February, 1956), 239.

in the following areas were the most evident; arithmetic computation, verbal problems in applied mathematics, problems in applications of percent, evaluation of use of simple formula, interpretation of graphic representation of data, and all phases of simple algebra that were tested. She emphasized the fact that the general mathematics students taking these tests were usually the best the representative schools had to send to the contest.

There does seem to be agreement concerning the characteristics of the low achiever even though there seems to be a variety of terms to name the same set of students. As Ogle writes in "Unfinished Revolution: Mathematics for Low Achievers," "what they are called is not so important as the acknowledgement of their existence."[16]

WHY A SPECIAL PROGRAM FOR THE LOW ACHIEVER?

We can find much current support in the literature for a mathematics program for the low achiever. As the previous section so clearly indicates, the low achiever has difficulties to overcome that the average or above average mathematics student does not have. The number of low achievers in our elementary and secondary schools is sufficient to warrant our immediate attention. A. M. Chandler, Director of the Experiences in Mathematical Ideas NCTM Project, wrote:

> It is estimated that at least thirty percent of our elementary and secondary school students are culturally

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[16] Ogle, loc. cit.

> deprived, socially disadvantaged, below average achievement, or educationally disadvantaged. Regardless of the specific name associated with these students, the fact is that approximately 16 million are slow learners. The concentration of these students is particularly noticeable in our largest cities. However, every school in rural and suburban America also has pupils who are not achieving as expected.[17]

"When President Johnson signed the 1.3 billion aid to education bill on April 11, 1965, he expressed hope that the bill would bridge the gap between helplessness and hope for more than five million educationally deprived children."[18] If we subscribe to the premise that "mathematics courses should be provided each year (K-12) for the low achiever,"[19] then the low achiever is mathematically deprived to the extent that we fail to provide such courses.

Sarah Greenholz,[20] writing in The Mathematics Teacher, listed several reasons why a program for the lower one-third of the school population is essential. Some of these are: students should master

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[17] A. M. Chandler, "Mathematics and the Low Achiever; Experiences in Mathematical Ideas (EMI) Project," Arithmetic Teacher, XVII (March, 1970), 196.

[18] F. Elder, "Mathematics for the Below Average Achiever in High School," The Mathematics Teacher, LX (March, 1967), 235.

[19] School Mathematics Study Group, Conference on Mathematics Education for Below Average Achievers (Stanford, California: Stanford University, 1964), p. 5.

[20] Sarah Greenholz, "Successful Practices in Teaching Mathematics to Low Achievers in Senior High School," The Mathematics Teacher, LX (April, 1967), 329-35.

skills of arithmetic to be self sustaining individuals in an automated technological society; competence in mathematics opens the door for pupils to pursue vocational training; and education is a continuous process today. Paul C. Rosenbloom, Director of the Minnesota School Mathematics and Science Center, wrote "mathematics and reading are the key subjects for making low ability children employable. If these children do not acquire abilities which make them employable, they become life long public charges."[21]

Both the U.S. Office of Education and the National Council of Teachers of Mathematics have shown an interest in better instructional programs in mathematics for the low achiever. The reasons for this interest have been summarized by Harry L. Phillips, Specialist in Mathematics Education for the U.S. Office of Education. The reasons include:

> 1. Lack of both achievement and interest in the instructional program have generally been shown to be the principal reasons for students' dislike of school.
>
> 2. Training in mathematics along with some degree of competence, gives a student a much broader choice of types of vocational training in the late secondary school or postsecondary technical training institutions.
>
> 3. The mathematical community and mathematics educator are very proud, and rightfully so, of the advancements made over the past 15 years in the

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[21] Paul C. Rosenbloom, "Implications of Psychological Research," The Low Achiever in Mathematics, OE 2906, Bulletin 1965, No. 31 (Report of a conference held in Washington, D.C., March 25-27, 1964, sponsored jointly by the U.S. Office of Education and the National Council of Teachers of Mathematics), p. 25.

> mathematics curriculums and in teaching at all levels of education. At all levels, however, the emphasis and attention have been directed toward the above-average mathematics achiever. We believe it is now time to show consideration for the low achiever.
>
> 4. The student of low general ability who is also likely to be a slow learner in mathematics, may, with the proper program and improved methods of teaching be able to enter the labor market less vulnerable to lurking unemployment possibilities.
>
> 5. Success or measurable achievement in mathematics has a close correlation with increased achievement in other disciplines.[22]

The low achiever today is receiving the same attention that the high achiever received after 1955.

WHAT MATHEMATICS FOR THE LOW ACHIEVER?

If we acknowledge the existence of the low achiever and that he needs a program in mathematics, the next logical question is, "what mathematics?" Since our concern is to make him a productive member of society, then it is appropriate that we consider, "what should be the mathematical competencies of every citizen?" Until most recently, the work of the Commission on Post War Plans created by the directors of the National Council of Teachers of Mathematics in 1944, has been a major influence on what mathematics competencies

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[22]Harry L. Phillips, "Why We Are Concerned About Low Achievers in Mathematics," The Low Achiever in Mathematics, OE 2906, Bulletin 1965, No. 31 (Report of a conference held in Washington, D.C., March 25-27, 1964, sponsored jointly by the U.S. Office of Education and the National Council of Teachers of Mathematics), pp. 2-3.

should be possessed by the average citizen. The essentials for functional competence in mathematics were placed as questions in a check list by the Post-War Commission. Because of the impact of the work of the Post War Commission on curriculum materials for the low achiever for the past twenty-eight years, the author chose to include these questions in detail.

The Check List[23]

1. Computation. Can you add, subtract, multiply, and divide effectively with whole numbers, common fractions, and decimals?

2. Percents. Can you use percents understandingly and accurately?

3. Ratio. Do you have a clear understanding of ratio?

4. Estimating. Before you perform a computation, do you estimate the result for the purpose of checking your answer?

5. Rounding numbers. Do you know the meaning of significant figures? Can you round numbers properly?

6. Tables. Can you find correct values in tables; e.g., interest and income tax?

7. Graphs. Can you read ordinary graphs: bar, line and circle graphs? the graph of a formula?

8. Statistics. Do you know the main guides that one should follow in collecting and interpreting data; can you use averages (mean median, mode); can you draw and interpret a graph?

9. The nature of a measurement. Do you know the meaning of a measurement, of a standard unit, or the

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[23]Raleigh Schorling (Chairman), "The Second Report of the Commission on Post-War Plans," The Mathematics Teacher, XXXVIII (May, 1945), 197-98.

largest permissible error, of tolerance, and of the statement that "a measurement is an approximation?"

10. Use of measuring devices. Can you use certain rulers (graduated to thirty-seconds, to tenths of an inch, and to millimeters), protractor, graph paper, tape, caliper micrometer, and thermometer?

11. Square root. Can you find the square root of a number by table, or by division?

12. Angles. Can you estimate, read, and construct an angle?

13. Geometric concepts. Do you have an understanding of point, line, angle, parallel lines, perpendicular lines, triangle (right, scalene, isosceles, and equilateral), parallelogram (including square and rectangle), trapezoid, circle, regular polygon, prism, cylinder, cone, and sphere?

14. The 3-4-5-relation. Can you use the Pythagorean relationship in a right triangle?

15. Constructions. Can you with ruler and compasses construct a circle, a square, and a rectangle, transfer a line segment and an angle, bisect a line segment and an angle, copy a triangle, divide a line segment into more than two equal parts, draw a tangent to a circle, and draw a geometric figure to scale?

16. Drawings. Can you read and interpret reasonably well, maps, floor plans, mechanical drawings, and blueprints? Can you find the distance between two points on a map?

17. Vectors. Do you understand the meaning of vector, and can you find the resultant of two forces?

18. Metric system. Do you know how to use the most important metric units (meter, centimeter, millimeter, kilometer, gram, kilogram)?

19. Conversion. In measuring length, area, volume, weight, time, temperature, angle, and speed, can you shift from one commonly used standard unit to another widely used standard unit; e.g., do you know the relation between year and foot, inch and centimeter, etc.?

20. Algebraic symbolism. Can you use letters to represent numbers; i.e., do you understand the symbolism of algebra--do you know the meaning of exponent and coefficient?

21. Formulas. Do you know the meaning of a formula--can you, for example, write an arithmetic rule as a formula, and can you substitute given values in order to find the value for a required unknown?

22. Signed numbers. Do you understand signed numbers and can you use them?

23. Using the axioms. Do you understand what you are doing when you use the axioms to change the form of a formula or when you find the value of an unknown in a simple equation?

24. Practical formulas. Do you know from memory certain widely used formulas relating to areas, volumes, and interest, and to distance, rate, and time?

25. Similar triangles and proportion. Do you understand the meaning of similar triangles, and do you know how to use the fact that in similar triangles the ratios of corresponding sides are equal? Can you manage a proportion?

26. Trigonometry. Do you know the meaning of tangent, sine, cosine? Can you develop their meanings by means of scale drawings?

27. First steps in business arithmetic. Are you mathematically conditioned for satisfactory adjustment to a first job in business; e.g., have you a start in understanding the keeping of a simple account, making change, and the arithmetic that illustrates the most common problems of communications and everyday affairs?

28. Stretching the dollar. Do you have a basis for dealing intelligently with the main problems of the consumer; e.g., the cost of borrowing money, insurance to secure adequate protection against the numerous hazards of life, the wise management of money, and buying with a given income so as to get good values as regards both quantity and quality?

29. Proceeding from hypothesis to conclusion. Can you analyze a statement in a newspaper and determine what is assumed, and whether the suggested conclusions really follow from the given facts or assumptions?

To determine whether schools were helping students attain the competencies as outlined in the check list, two studies were conducted by Davis and Beckmann.

Davis[24] designed a 90-question test to measure what he called functional competence in mathematics. His intent was to measure the objectives in each of the 29 competencies of the check list; however, each item in the test was not assigned to a specific competency so it is not possible to equate specific questions with specific competencies. The test was administered to 2523 pupils, who were seniors, in various schools in the state of Michigan. The most general and obvious conclusion that could be drawn from the study was that the vast majority of the students were leaving high school with an inadequate understanding and command of the essentials for functional competence in mathematics.

Bechmann[25] also constructed a test, Beckmann's Mathematical Literacy Test, to measure the level of competency as defined by the

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[24]David John Davis, "A Comparative Study of Achievement Levels of Twelfth Grade Pupils on a Test Designed to Measure Functional Competence in Mathematics" (unpublished Doctor's dissertation, University of Michigan, Ann Arbor, 1950).

[25]Milton W. Beckmann, "The Level of Mathematical Competency and Relative Gains in Competency of Pupils Enrolled in Algebra and General Mathematics" (unpublished Doctor's dissertation, University of Nebraska, 1951).

29 competencies suggested by the Commission on Post War Plans, and the relative gains in mastery of such competencies as a result of the study of algebra and general mathematics in the nith grade. Beckmann's Mathematical Literacy Test was designed so that particular questions would measure the attainment of a specific competency. The test was given as a pretest in November and as a posttest in April to students in 42 high schools in Nebraska. Seventeen of the schools, with 562 students participating in the study, required ninth graders to take algebra, and 25 schools, with 734 students participating, required ninth graders to take general mathematics. Analysis of the data revealed a low level of mathematical literacy (understanding and mastery of the 29 competencies) of ninth grade students enrolled in either algebra or general mathematics classes and very small point gains after a year of study in the ninth grade. When the mean gains for general math classes and algebra classes were compared, it was found the general math classes had a significantly higher (0.01 level) gains in achievement than the algebra classes.

Since the publication of the check list and the previously mentioned studies, many changes have been introduced into mathematics. In fact, "by 1961 the reform in mathematics education was of sufficient proportions to be labeled officially a revolution by the NCTM."[26]

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[26] A History of Mathematics Education in the United States and Canada, Thirty-second Yearbook of the National Council of Teachers of Mathematics (Washington, D.C.: The Council, 1970), p. 281.

Lucien B. Kinney[27] noted the following shortcomings in the curriculum projects that were a part of this "revolution."

> 1. Disregard of the Purpose of Secondary Education. Data on what can be taught, need to be supplemented by information on what should be taught, in light of the purposes of the secondary schools, before the program of curriculum development can be completed.
>
> 2. Neglect of Important Concomitant Outcomes. While discovery and generalization in the area of abstract mathematics are encouraged, the development of systematically applied techniques of problem solving, in the sense advocated, for example by Polya, is neglected.
>
> 3. Neglect of Differential Needs of Various Pupil Groups. Provision for the slower pupils on the other hand, is based on the premise that, given more time and a less formal approach, slower pupils can cover the same materials as the more rapid pupils. This is in contradiction to research findings as well as to the experience of classroom teachers.

Most of the curriculum projects of the late 50's and 60's were concerned with the college capable student and were not with the low achiever. However, the result of men such as Morris Kline[28] and Lucien B. Kinney directing our attention to some of the shortcomings of the "revolution," has been a recognition that there is a real need to reconsider what should be the mathematical skills and competencies essential for enlightened citizens. Unless we decide what mathematics should be taught, and not just to college capable

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[27]Lucien B. Kinney, "Mathematics," The High School Curriculum, ed. Harl R. Douglass (New York: Ronald Press Company, 1964), pp. 353-54.

[28]Benjamin DeMott, "The Math Wars," American Scholar, XXXI (Spring, 1962), 296-310.

students, it will be difficult to develop better materials or methods for helping the low achiever overcome his deficiencies.

Recognizing this need, the National Council of Teachers of Mathematics established the Committee on Basic Mathematical Competencies and Skills in 1970. The committee was composed of Eugene D. Nichols, Glyn H. Sharpe, and E. L. Edwards, Chairman. This committee has prepared a preliminary report, "Mathematics Competencies and Skills Essential for Enlightened Citizens," to be published in the fall of 1972 in The Mathematics Teacher. In all probability, this report will eventually evolve into the basis for much of the future curriculum development for the low achiever.

Again the author would like to stress the fact that the time between the issuance of the twenty-nine questions by the Commission on Post-War Plans and the present has primarily been devoted to developing programs for the college capable students and have not stressed mathematics for everyday living. With the emphasis in mathematics education changing to the low achiever, it is imperative that we redefine what mathematical competencies and skills are needed to function as an enlightened citizen.

RESEARCH RELATIVE TO METHODS OF TEACHING THE LOW ACHIEVER

Assuming the low achiever has special needs that must be met by special programs, it is appropriate to investigate what methods are suggested for use with the low achiever and does research support

these suggestions. Lauren G. Woodby[29] included in his report of a conference held in Washington, D.C., March 25-27, 1964, on the low achiever in mathematics, suggested guidelines for teaching mathematics to the low achiever. These included such suggestions as:

1. Modern educational technology should be exploited.
2. Classroom activities should be both purposeful and varied.
3. Particularly for the low achiever, the need for mathematics comes from experiences in the physical world.
4. The teacher should be receptive to questions.
5. A laboratory setting is especially effective for low achievers.

Additional suggestions are found in Today's Education where Greenholz[30] listed the following techniques to be used with the low achiever:

1. Let the pupils succeed.
2. Use warm up exercises to get pupils settled down so that the day's lesson can start.
3. Accustom pupils to working out verbal problems.
4. Use a overhead projector when explaining a procedure.

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[29]Woodby, loc. cit.

[30]S. B. Greenholz, "Reaching Low Achievers in High School Mathematics," Today's Education, LVII (September, 1968), 70-72.

5. Give the class an opportunity to learn through several senses.
6. When many pupils appear to need the same kind of help, use supervised group practice at the board.
7. Make each daily lesson complete in itself.
8. Usually change the activity every 15 to 20 minutes.

Writing in The Mathematics Teacher concerning the low achiever, Greenholz said, "If at all possible put smaller classes at the top of your list of priorities. Large classes with little equipment for the low achiever results in expensive education in the long run."[31]

A search through ERIC reveals several reports on low achievers but in almost all cases these are merely descriptive. Much of this descriptive research is "advice from the firing line" variety. It consists of people sharing with others techniques which have successfully worked with their students. Other articles have been written to inspire teachers to motivate their students to learn. Because of missing data and a lack of controls, any conclusions drawn from these efforts would be somewhat dubious.[32] The comments were made by Herriot in 1967, in a School Mathematics Study Group

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[31]Greenholz, "Successful Practices in Teaching Mathematics to Low Achievers in Senior High School," loc. cit.

[32]Sarah T. Herriot, "SMSG Report No. 5, The Slow Learner Project: The Secondary School 'Slow Learner' in Mathematics" (Educational Information Center (ERIC) Report Number ED 021755), pp. 2 and 7.

report on the low achiever. However, since that time some research has been done relative to effective teaching techniques to be used with the low achiever.

Newman and Seiser[33] reported on a study conducted in six junior high schools that qualified for ESEA funds. Each school was allowed funds for one extra mathematics teacher. This teacher would offer tutorial help and remedial opportunities to pupils in small ungraded groups. With some pupils, the emphasis was on supplementary enrichment activity to increase interest. Six other comparable junior highs were chosen to act as control schools. Five out of six of the junior highs which had a floating teacher showed significant (0.05 level) improvement in arithmetic skills while only one out of six of the control schools showed significant improvement. Pupil attitude toward mathematics was significantly (0.05 level) better than it was reported last semester for those in project schools as contrasted to no appreciable improvement in attitude toward mathematics in the control schools. This study involving a "floating teacher" was the only one to bring into the regular classroom additional personnel to work with the low achiever.

Concern over class size led Maden[34] to conduct a study comparing

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[33] T. B. Newman and W. Seiser, "Floating Teacher; Help for the Mathematically Disadvantaged," The Mathematics Teacher, LX (November, 1967), 753-55.

[34] J. V. Madden, "Experimental Study of Student Achievement in Relation to Class Size," School Science and Mathematics, LXVIII (October, 1968), 619-22.

student achievement in general mathematics classes consisting of seventy to eighty-five students with classes consisting of from twenty-five to forty students. A pretest and posttest, using the Contemporary Mathematics Test, Junior High School Level, Forms X and W, were administered. Using analysis of variance, no significant differences were found on the pretest; however, significant differences (0.05 level) were found on the posttest with the large group general mathematics classes doing better thatn the samll groups.

Some educators are suggesting that we can teach the low achiever modified algebra. A study comparing achievement and attitude of ninth grade pupils in general mathematics, modified algebra, and reading was conducted by Sederberg.[35] When the three groups were compared on arithmetic computation by analysis of variance, a significant F-ratio was obtained with the general mathematics group having the highest mean scores. There was a slight improvement in attitude in the general mathematics group while there was a slight regression in the modified algebra. The reading group showed a slight increase in computational skills indicating factors other than formal mathematics instruction contributed to the learning of skills and concepts in mathematics.

Another study involved the use of the language of algebra to describe relationships in the world in which we live. A ninth grade

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[35]C. H. Sederberg, "Comparison of Mathematics Teaching Methods for Average and Below Average Ninth Grade Pupils," Journal of Educational Research, LIX (July, 1966), 435-40.

general mathematics class, made up of lowest ability students from a school of 1600, used experiments to study mathematical concepts. All the work was based upon the notion that the focal point would be the use of the language of algebra to describe relationships in the world in which we live. No data were collected concerning achievement or attitude. However, the author felt the students were able to describe many relationships discovered through experimentation by algebraic equations. Some suggestions for the teacher of the low achiever that came from this study were: approach all students as if they can learn; repeat mathematical ideas in a variety of settings; and free the students so that they can think as best they can.[36]

A modified programmed lecture and mathematical games approach to teaching ninth grade general mathematics students was tried in Lafayett Parish School System. Two classes of twenty students each were divided into two groups, IQ below 85 called low achiever and IQ above 85 called underachievers. One group received a modified programmed lecture approach and mathematical games while the control used a textbook. The use of modified programmed lecture approach and mathematical games resulted in significant achievement and

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[36]H. Fremont and N. Ehrenberg, "Hiddent Potential of Low Achievers," The Mathematics Teacher, LIX (October, 1966), 551-57.

attitude gains with no differences found between IQ levels.[37] We might ask, what is a mathematics laboratory? The Central Iowa Low Achiever Motivational Project offers a good answer.

> Primarily a mathematics laboratory is a state of mind. It is characterized by a questioning atmosphere and a continuous involvement with problem solving situations. Emphasis is placed upon discovery resulting from student experimentation. The teacher acts as a catalyst in the activity between students and knowlege.
>
> Secondarily, a mathematics laboratory is a physical plant equipped with such material objects as calculators, overhead and opaque projectors, filmstrips, movies, tape-recorders, measuring devices, geoboards, solids, graph boards, tachistoscopes, construction devices, etc. Since a student learns by doing, the lab is designed to give him the objects with which he can do and learn.
>
> The primary goal of the lab approach is to change the student's attitude toward mathematics. Most students have become so embittered by habitual failure that they hate mathematics and everything connected with it. There is little possibility of this student learning mathematics until an attitude change has been effected. It is because of this goal that our approach is different. Some would label our approach as "fun and games," but I am sure that close examination will bring realization that everything in the program is oriented towards the twin goals of attitude change and mathematical improvement. Learning can be enjoyable and in fact, should be so. Once the student sees that he can enjoy mathematics, he will want to learn about it. Then and only then is learning possible.[38]

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[37] Thomas Jones, "Effect of Modified Programmed Lectures and Mathematical Games Upon Achievement and Attitude of Ninth Grade Low Achievers in Mathematics," The Mathematics Teacher, LXI (October, 1968), 603-607.

[38] Joseph T. Zimmerman, "LAMP Booklet," (Des Moines, Iowa: Des Moines Public Schools, 1968), p. 3.

Schippert[39] compared discovery teaching by the use of verbal or written descriptions of models of mathematical principles (abstract method) with manipulation of actual models or representations of mathematical principles (laboratory method). Both groups used the School Mathematics Study Group textbook, Volume I, Part I. The entire first semester seventh grade of an inner city junior high school was selected. One-half of the group was taught by the laboratory method and one-half by the abstract method. The Iowa Every Pupil Test of Basic Skills, Test D: Arithmetic and Duttons Attitude Scale Toward Arithmetic were given as pretests and a parallel form of the arithmetic test and the same attitude test were given five months later as a posttest. To check for retention, the tests were given two and one-half years later. Analysis of covariance on the immediate and delayed scores was used to check for differences in means. The following results were obtained:

1. Statistically significant differences in favor of the groups taught by the laboratory method was obtained in growth of arithmetic skills.
2. A significant difference was found between the groups of high achievers, average achievers, and low achievers, as listed, in growth of arithmetic skills.

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[39]Frederick Arthur Schippert, "A Comparative Study of Two Methods of Arithmetic Instruction in an Inner-City Junior High School," (Wayne State University, 1964), Dissertation Abstract, 25:5162-163, March, 1965.

3. No significant difference was found among any of the factors or interactions in improving opinion toward arithmetic.

Another alternative to grouping by intact classes would be to group by content topics. R. E. Willcutt,[40] writing in the Journal of Educational Research, indicated there were arguments that could be advanced in favor of content grouping. A grouping procedure that allows for flexibility and change would create positive attitudes on the part of the students toward mathematics. Flexible groups with different programs for each grouping level would increase the achievement level of all students. The author did not find any studies involving flexible grouping but feels this alternative should be explored for meeting the needs of the low achiever.

RESEARCH RELATIVE TO MATERIALS FOR THE LOW ACHIEVER

Wolfson, when writing about problems of change in the classroom, wrote,

> I asked the teachers, as we worked together to describe some of the problems they met and some of the concerns they had about how their class was operating. Probably the most frequent concern was the problem of getting an adequate amount and variety of materials in the interest areas

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[40] R. W. Willcutt, "Ability Grouping by Content Topics in Junior High School Mathematics," Journal of Educational Research, LXIII, 152-56.

> that the children selected and at appropriate levels of difficulty.[41]

Rosskoph and Kaplan, when discussing the training of specialists in teaching children from disadvantaged areas, wrote, "for the purpose of teaching disadvantaged children the interns find almost all textbooks inadequate and inappropriate."[42]

Woodby[43] listed several guidelines for the preparation of materials for the low achiever. Some of these were: materials should provide for the development of understandings essential for vocational competence; opportunity for success should be a major aim in the design of the learning materials; materials should provide a varied approach to the development of mathematical concepts; the learning materials should be graded in content; evaluation of the experimental materials should be conducted in a variety of schools; materials should be tried out with different types of low achievers; appropriate measuring instruments should be designed and constructed to determine the degree to which the new materials accomplish their purpose. The writer would like to emphasize the need to evaluate materials developed for the low achiever. It is possible to find a

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[41]Bernice J. Wolfson, "Teaching for Individuality and Personal Learning," Individualization of Instruction A Teaching Strategy (New York: Macmillan Company, 1970), p. 117.

[42]M. F. Rosskoph and J. D. Kaplan, "Educating Mathematics Specialist to Teach Children from Disadvantaged Areas," The Arithmetic Teacher, XV (November, 1968), 608.

[43]Woodby, loc. cit. The writer of this study underlined these sentences for emphasis.

number of groups preparing materials to be used with the low achiever, but it becomes difficult to find a systematic evaluation of the effectiveness of these materials in improving student achievement and student attitude.

Of prime concern, in the construction of materials for the low achiever, should be the development of materials that require active involvement on the part of the learner. In a report of an eight-week workshop held at Temple University in 1968, A. M. Bachmann indicated that a major premise in preparing materials for the low achiever is that "the students (low achievers) actively participate in the lessons."[44] Howell, in his report on Project SOS (Save Our Slow Ones) wrote, "throughout the project, the hypothesis was: a student will understand a mathematical concept better if he has a visual representation or a concrete object at hand when the concept is introduced."[45] In an article by Keys[46] in The Arithmetic Teacher we find four statements from learning theory that lend support to the premise that the learner needs to be actively involved in the learning process. They are: learning is based on experience; learning

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[44]A. M. Bachmann, "Some Suggestions for Motivating Students in General Mathematics Classes," The Mathematics Teacher, LXIV (November, 1971) 659.

[45]D. Howell, "Project SOS (Save Our Slow Ones)," The Arithmetic Teacher, XIX (January, 1972), 29-30.

[46]R. E. Keys, "Considerations for Teachers Using Manipulative Materials," The Arithmetic Teacher, XVIII (December, 1971), 551-58.

is enhanced by motivation; learning proceeds from concrete to abstract; and learning requires active participation on the part of the learner.

One method of obtaining active involvement on the part of the learner is through the use of mathematical games. Lola May has made extensive use of games in her teaching and writes, "dice and card games in the classroom. Why not? They're good teaching tools for math. Games can be created to reinforce skills that have already been learned, or are being learned in mathematics."[47]

Burgess[48] conducted a study to determine whether a strategy involving regular usage of mathematical games could prove effective for teaching mathematics to low achieving secondary students. Text materials and math games used in the study were developed for teaching concepts and skills with rational numbers in an ESEA Title III Project in Palm Beach County Florida schools. Paper and pencil activity sheets related to the texts were developed for use by the control group. Ten teachers in nine public and parochial schools each taught the same number of classes in the experimental and control groups.

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[47]Lola May, "Math Games," Grade Teacher, LXXXVIII (September, 1970), 70.

[48]Ernest Burgess, "A Study of the Effectiveness of the Planned Usage of Mathematical Games on Learning of Skills and Concepts and on the Attitude Toward Mathematics and the Learning of Mathematics of Low Achieving Secondary Students" (The Florida State University, 1969), Dissertation Abstract, 30A:5333-334.

Twelve classes were randomly assigned to each for a total of 248 subjects in the experimental group and 240 in the control. Each received one-half a period per day of normal instruction and one-half a period per day of game or activity sheets, for eight weeks. Pretesting involved the SCAT, two twenty-item achievement tests and a semantic differential attitude scale. After four weeks two similar achievement tests were administered. The attitude scale was administered after four weeks and again after eight weeks. Using analysis of covariance, both posttreatment attitude measures yielded significant differences in mean scores strongly favoring the experimental treatment. However, on achievement, significant differences were found favoring the control treatment.

In order to determine how extensive mathematical games were being used by schools, Brandes[49] mailed fifty questionnaires to fifty persons selected at random from 100, requesting copies of selected recreational materials. Forty-two questionnaires were returned and from these Brandes found: including materials as part of regular lesson plans was the method selected most often; teachers felt that the use of recreational materials improved their classes and resulted in stimulating interest; and games were most useful in ninth and tenth grade subjects.

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[49]Louis Brandes, "Using Recreational Mathematics Materials in the Classroom," The Mathematics Teacher, XLVI (May, 1953), 326-29.

In most articles dealing with the preparation of materials for the low achievers, some reference to their reading ability is usually made. When reviewing the literature concerning the characteristics of the low achiever, it was often the case that low achievers were below grade level in reading as well as mathematics.[50] The implications for preparing written materials for the low achiever are obvious. Barney, Professor of Education at Northern Illinois University, explored some of the reading problems that should be given special attention in the mathematics class.

> Technical Words. Avoid assigning youngsters who have general reading difficulties those mathematical problems that contain too many technical or difficult words. Five percent unfamiliarity may be too many.
>
> Problem Length. There are two factors that affect general readability of narrative mathematics problems: word length and sentence length.
>
> Occasional Vocabulary Words. Meanings should be clarified before the assignment is given.
>
> Words with Multiple Meanings. The teacher should be aware of problems that contain words having one meaning in literature and another in arithmetic.
>
> Narratives with Unnecessary or Insufficient Data. Problems containing insufficient data or unnecessary information are valuable, but they should be used with discretion to determine how well the child is able to use his reading skills at the interpretive or analytical level.
>
> Student Restatement. Allow students the opportunity to restate their problems in their own words. But note that

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[50] Sister Agnes Jerome, "A Study of Twenty Slow Learners," Journal of Educational Research, LIII (September, 1959), 23-27.

the problem restatement should be oral, not written.

Punctuation. Punctuation should be retaught and reapplied as it refers to mathematics notation, for many marks used by mathematicians have no counterpart in meaning in the field of literature.

Abbreviations. Mathematics teachers often expect children to understand the meanings of abbreviations without benefit of additional instruction.[51]

It would seem that the person preparing materials for the low achiever and the person teaching the low achiever should have skills in the teaching of reading. For as Barney writes, "the efficiency of a mathematics student can be increased if he is given assistance in learning to read problems."[52]

Some research is being done on the effects of improved reading skills on achievement. Damer[53] conducted a study in which twelve tenth grade students, classified as underachievers, received instruction in reading for five months. At the end of that time, the students showed a seven months' increase in mathematics scores.

Curry[54] investigated the effects of reading instruction upon achievement in seventh grade arithmetic and found no significant

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[51] L. Barney, "Problems Associated with the Reading of Arithmetic," The Arithmetic Teacher, XIX (February, 1972), 131.

[52] Ibid.

[53] Dan Damer, Self, Contained, Reading Oriented Classes in Secondary Schools (ERIC Document No. ED 041 705, May 8, 1970).

[54] John Curry, "The Effect of Reading Instruction Upon Achievement in Seventh-Grade Arithmetic" (Indiana University, 1959), Dissertation Abstract, 15:2059, November, 1955.

differences between groups which had or did not have specific reading instruction. Troxel[55] found the ability to read mathematical materials appeared related to general reading ability. However, practice without instruction did not result in better speed and comprehension scores.

SUMMARY

The author, in his review of literature, has answered the following questions: Who are the low achievers? What are his characteristics? Why a special program for the low achiever? What mathematics for the low achiever? What instructional methods should be used with the low achiever? What materials should be provided for the low achiever? Several conclusions can be drawn from the review of literature.

1. The term "low achiever" is usually applied to those students who score below grade level on a standardized mathematics test.
2. The low achiever in general has a history of failure, negative attitude toward mathematics and school in general, short attention span, poor self image; low reading scores; and poor motivation in mathematics.
3. Students become more consistently identified as low achievers after leaving the sixth grade.

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[55]Vernon Troxel, "Reading Eighth Grade Mathematical Materials for Selected Purposes" (University of Illinois, 1959), Dissertation Abstract, 20:168-69, July, 1959.

4. The number of students classified as low achievers is sufficiently high to necessitate special programs if we assume there are basic competencies needed by all enlightened citizens.
5. Low achievers need a chance to succeed, a variety of learning activities, laboratory activities, active involvement in the learning process, and individual help.
6. Materials need to be developed for the low achiever that are activity oriented; are at the reading level the student can understand; allow for short attention span; and help develop basic skills in operations with whole numbers, fractions, decimals, and percent.

Unless the reader feels the importance of the teacher has not been recognized, suffice it to say that the statement, "the sine qua non in a course for the low achiever is the skilled teacher who is well qualified in mathematics and interested in working with the low achiever,"[56] is readily endorsed by the author.

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[56] Ogle, loc. cit.

CHAPTER III

PROCEDURE

This study involved an evaluation of materials developed by participants in a National Science Foundation sponsored Summer Conference for Teachers of Low Achievers in Mathematics held in the summer of 1971, at the University of Nebraska under the direction of Dr. Milton W. Beckmann assisted by Mr. Jack Beal. Twenty-four teachers from eleven states as well as American Samoa participated in the three-week conference.

DEVELOPMENT OF CONFERENCE MATERIALS

One of the goals of the conference was the preparation of materials to be used by teachers of low achievers. Since teachers from all secondary grade levels (7-12) would be present, it was necessary to develop materials that would not be organized around grade levels. Both the author and Dr. Beckmann realized that the classroom teacher is limited by time and money in the search for and selection of materials to be used in the classroom. All of these considerations led the author, under the supervision of the Director, to develop the following guidelines to be used in the writing of materials for the reluctant learner.

1. Materials should be organized around units based upon competencies that all enlightened citizens should possess.

2. Behavioral objectives should be used to describe the type of student behavior that the materials could be expected to produce.

3. The materials should stress student involvement.

4. Extensive use should be made of mathematical games and puzzles (both commercially prepared and teacher prepared).

5. A variety of activities should be provided for each competency so that the teacher could select those that best meet his individual needs.

6. Materials should be developed that could be used by large groups, small groups, and/or individuals.

7. Participants should be allowed to select those competencies for which they wished to write units.

In order to meet the above goals, the following format was selected to be used in organizing and writing the units of instruction.

Competency:

Objectives:

Activity 1

Suggested Materials:

Directions for Students:

Suggested Strategies:

Activity 2

Suggested Materials:

Directions for Students:

Suggested Strategies:

A sample unit may be found in Appendix A. The reader will note the first part of the unit is a statement of the competency to be developed through the use of the suggested activities. These competencies were selected from those proposed by the Committee on Basic Mathematical Competencies of the National Council of Teachers of Mathematics. A complete listing of the competencies for which materials were prepared may be found in Table II. A classroom teacher who has a student experiencing difficulty with a particular competency needs a source of materials indexed in such a way that it takes a minimum amount of time to locate activities designed to help the student achieve this competency. By having the competency and specific behavioral objectives listed first, the teacher was enabled to meet this need. The section titled "suggested materials" was included so that the teacher might quickly decide whether appropriate materials for the activity were on hand. If not, he might proceed to the next activity under that competency. The directions to the student were written using a vocabulary that recognized that the low achiever quite often has a low reading level. Also it was felt the teacher would be encouraged to use the materials with individual students if the directions were already written. Finally, realizing that activities involving a large number of students might be new for some mathematics teachers, it was the opinion of the writer that some suggested strategies for using the activity should be included in the materials.

TABLE II

BASIC MATHEMATICAL COMPETENCIES FOR WHICH CONFERENCE MATERIALS WERE DEVELOPED

Competency/Objective
1. Ability to perceive patterns displayed by means of sequences of specific instances.
2. Describe a given positive rational number using decimal, percent, or fractional notation.
3. Write an equivalent fraction for given fractions, such as 1/2, 2/3, and 3/5.
4. Use the standard algorithms for the operations of arithmetic of while numbers.
5. Solve addition, subtraction, multiplication and division problems with fractions having denominators less than twenty.
6. Solve addition, subtraction, multiplication, and division problems with decimal fractions.
7. Construct a true mathematical statement from a given verbal problem.
8. Solve simple linear equations.
9. Construct a graph indicating the relationship of two variables from a given set of data.
10. Describe parallel lines, perpendicular lines, and intersecting lines using drawings of intuitive concepts.
11. Classify simple plane figures by distinguishing some of their properties.
12. Compute the perimeter of a given polygon.
13. Compute the area of a rectangle and of a triangle.
14. Construct bisectors of lines and angles.
15. Identify the conditions for similarity of triangles and use the properties of similarity to solve problems.

TABLE II (continued)

16.	Classify solid figures by distinguishing some of their properties.
17.	Apply the common English measures of length, volume, weight, time, money, and temperatures.
18.	Convert from one measure to an equivalent one with larger or smaller units in the English System.
19.	Convert, using tables, English to metric measure and conversely.
20.	Recognize that no measurement is precise.
21.	Use metric units of length, mass and volume in making measurements.
22.	Use standard measuring devices of length, area, volume, time, and temperature to make measurements.
23.	Round off measurements to the nearest unit of the measuring device used such as rulers, protractors, and thermometers.
24.	Predict the probability of simple events occurring.
25.	Plan a budget including record keeping of personal and travel expenses.
26.	Write simple sentences showing the relation =, <, >, for two given numbers.

SELECTION OF TEACHERS

All twenty-four teachers attending the conference were invited to participate in an evaluation of the materials during the fall semester of the 1971-72 school year. Each teacher who agreed to participate was asked to complete an information sheet (Appendix C).

Of the twenty-four members of the summer conference, ten

agreed to participate in the study. Although most members wanted to participate, fourteen could not for these varying reasons: the inability to randomly select students, not directly involved in enough classes of low achievers, and lack of finances. Also one conference member had three additional teachers who would agree to participate and one conference member agreed to teach three experimental and three control classes. Thus a total of thirteen different teachers from ten different schools in six states agreed to teach fifteen experimental and fifteen control classes. A listing of the school participating in the study may be found in Table III. Each teacher, participating in the study, agreed to:

1. teach a minimum of one experimental and one control class.
2. use at least six units during the fall semester with the experimental class.
3. where possible, to use the units rather than any textbook materials when teaching the experimental class.
4. submit a written evaluation of each unit used.
5. administer an attitude and an achievement test to both the experimental and the control classes.

A sample of the "Directions for Evaluation of the Units" mailed to each teacher may be found in Appendix C.

SELECTION OF STUDENTS

In order to provide as near a true experimental design as possible, it was decided to assign the students to experimental and

TABLE III

SCHOOLS PARTICIPATING IN STUDY

School	Public/ Private	City	State
Westlane Junior High	Public	Indianapolis	Indiana
Bishop Hartley High	Private	Columbus	Ohio
New Hartford Central	Public	New Hartford	New York
Maple Heights West Junior High	Public	Maple Heights	Ohio
Prince Edward Academy	Private	Farnville	Virginia
Elkhorn High	Public	Tilden	Nebraska
Marian High	Private	Cincinnati	Ohio
Lincoln Junior High	Public	Park Ridge	Illinois
Technical High	Public	Omaha	Nebraska
Carl Sandburg Middle School	Public	Freeport	Illinois

control classes by a table of random numbers if they were not assigned by computer (computer assignment resulting in random assignment). Of the fifteen pairs of experimental and control classes, students were assigned to ten of them by random numbers and to five of them by computers. There were three pairs of seventh grade classes, four pairs of eighth grade classes, six pairs of ninth grade classes, and two pairs of twelfth grade classes. Three of the schools used in the study were private and seven were public. A total of 263 students

used the conference materials while 254 were assigned to control classes. Table IV is a detailed listing of classes by size, grade level, and method of assignment of students.

METHODS OF INSTRUCTION

Experimental Class

As mentioned in an earlier part of the chapter, each teacher agreed to use at least six units during the fall semester in the experimental class. The teachers were allowed to select the units and activities they wanted to use. This was necessary because of the variety of classes being taught by teachers in the study. In all cases, the teachers chose to use the conference materials in conjunction with the regular classroom work. This was completely consistent with the philosophy of providing the teacher with materials that could be used to supplement rather than to replace regular classroom materials.

Control Class

Participating teachers were asked to use the same procedures and materials with the control class that were used with the experimental class except that no conference materials were to be given to the control class. Also, teachers were asked to maintain a record of the activities with which the control students were involved while the experimental students were using the conference materials. Every attempt to control all variables, except for the use of

TABLE IV

NUMBER OF STUDENTS, METHOD OF ASSIGNMENT AND GRADE LEVEL OF VARIOUS CLASSES PARTICIPATING IN THE STUDY

School	Number of Students		Method of Assignment	Grade Level
	Experimental	Control		
Westlane Junior High	20	17	Computer	9
Bishop Hartley High	32	32	Computer	9
New Hartford Central	10	14	Random	8
New Hartford Central	18	13	Random	9 (Business Math)
New Hartford Central	20	14	Random	9 (General Math)
Maple Heights West Jr. High	27	26	Computer	7
Prince Edward Academy	18	18	Random	12
Elkhorn	6	18	Random	9
Marian High	17	11	Computer	9
Lincoln Junior High	23	31	Random	8
Technical High	19	7	Computer	12
Carl Sandburg Middle School	15	14	Random	8
Carl Sandburg Middle School	13	14	Random	7
Carl Sandburg Middle School	12	12	Random	7
Carl Sandburg Middle School	13	12	Random	8

conference materials, was made.

CONSTRUCTION OF AN ACHIEVEMENT TEST

C. A. Riedesel, writing in the Arithmetic Teacher,stated that ". . . instruments for measurement should reflect the objectives of the study. Too often a readily available test is used to measure many different types of abilities. Nonappropriate standardized measures should be avoided."[1] Believing we often attempt to measure specific objectives with very general achievement tests, the author chose to construction an achievement test for the twenty-six competencies included in the summer conference materials.

The writer developed a 52-item multiple choice test with five alternatives for each question. Two items were selected for each competency with the assumption that the probability of guessing two correct answers would only be 0.04, certainly small enough to conclude the examinee had the competency.

The first step in the construction of the test was the selection of test items. Sixty-nine test questions were initially written by the author. A matrix listing the competency or competencies that each test item measured was constructed. These test items were then submitted to a panel of judges (Table V) who were asked

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[1]C. A. Riedesel, "Some Comments on Developing Proper Instrumentation for Research Studies in Methematics," Arithmetic Teacher, XV (Fall, 1968), 165.

TABLE V

ITEM JUDGES FOR ACHIEVEMENT TEST

Judge	Position	School
Dr. Milton W. Beckmann	Professor of Secondary Education	University of Nebraska Lincoln, Nebraska
Dr. Buren Thomas	Mathematics Department Chairman	Northeast High Lincoln, Nebraska
Dr. Ronald Massie	Mathematics Coordinator (K-12)	Lincoln Public Schools Lincoln, Nebraska
Mr. Jerry Beckman	Mathematics Department Chairman	Lincoln East Lincoln, Nebraska
Miss Barbara Wickless	Mathematics Supervisor	University of Nebraska Lincoln, Nebraska

to check their validity and rate them as good, usable, or unacceptable. Using the results of this panel, an achievement test consisting of 71 items was written. The additional items were added as the result of various comments written by the judges. Two forms of the test were prepared, one form having the order of the items reversed so that every item would have approximately the same number of responses. The items were reversed so that the test could be administered in one hour (alternate forms being given to every other student) and still obtain results on all 71 items.

A total of 169 seventh, eighth, and ninth grade students from Lincoln Northeast and Lincoln East High School were given either Form A or Form B in order that an item analysis could be performed.

In Table VI the reader will find the distribution of responses, the mean score of all those who failed the item (Q-mean), the mean score of all those who passed the item (P-mean), the ratio of number failed to total (Q-ratio), and the ratio of number passed to total (P-ratio) for the combined totals from Form A and Form B.

TABLE VI

ITEM ANALYSIS OF COMBINED SCORES
FROM FORM A AND FORM B

Item	Key	Alternatives						Q-Mean	P-Mean	Q-Ratio	P-Ratio
		a	b	c	d	e	Omit				
1	2	1	41	16	21	21	69	23.81	32.85	0.757	0.243
2	5	4	5	5	2	6	147	25.31	44.82	0.964	0.036
3	3	80	1	10	2	9	67	25.88	28.00	0.941	0.059
4	1	86	10	5	2	2	64	21.25	30.59	0.991	0.509
5	4	0	4	5	93	8	59	22.36	28.99	0.450	0.550
6	2	6	86	4	3	8	62	22.76	29.14	0.491	0.509
7	5	7	15	2	5	81	59	23.08	29.19	0.521	0.479
8	1	68	11	3	5	26	56	23.44	29.82	0.598	0.402
9	5	11	9	3	4	83	59	22.17	29.98	0.509	0.491
10	4	5	3	13	94	1	53	20.51	30.39	0.444	0.556
11	4	12	21	5	46	29	56	24.33	30.48	0.728	0.272
12	3	8	8	56	7	32	58	22.42	33.25	0.669	0.331
13	2	4	85	4	14	11	51	21.94	30.02	0.497	0.503
14	5	6	13	30	3	68	49	22.68	30.94	0.598	0.402
15	1	82	10	7	4	20	46	21.23	31.07	0.515	0.485
16	4	3	8	11	76	26	45	21.60	31.39	0.550	0.450
17	1	23	40	19	5	24	58	24.57	35.13	0.864	0.136
18	1	85	14	18	5	3	44	21.98	29.99	0.497	0.503
19	4	10	38	10	59	8	44	22.75	32.08	0.651	0.349
20	1	49	10	42	5	15	48	24.07	30.76	0.710	0.290
21	4	11	17	20	33	41	47	24.83	30.85	0.895	0.195
22	5	10	31	8	6	74	40	22.68	30.27	0.562	0.438
23	1	21	40	17	17	36	38	25.26	31.24	0.876	0.124
24	3	13	8	66	22	28	32	23.27	30.27	0.609	0.391
25	2	8	42	30	37	20	32	23.84	32.55	0.751	0.249
26	2	44	34	15	12	31	33	25.56	27.76	0.799	0.201
27	2	4	114	4	18	11	18	22.40	27.75	0.325	0.675

TABLE VI (continued)

Item	Key	Alternatives a	b	c	d	e	Omit	Q-Mean	P-Mean	Q-Ratio	P-Ratio
28	3	13	28	74	3	32	19	22.37	30.68	0.562	0.438
29	2	56	17	10	10	38	38	25.70	28.71	0.899	0.101
30	3	3	13	108	10	17	18	20.26	29.25	0.361	0.639
31	4	93	14	5	9	29	19	25.64	32.56	0.947	0.053
32	5	30	29	19	32	20	39	25.62	28.90	0.882	0.118
33	3	40	6	83	7	25	8	21.60	30.57	0.509	0.491
34	3	11	1	94	3	50	10	22.27	28.99	0.444	0.556
35	3	4	5	121	9	21	9	19.35	28.64	0.284	0.716
36	4	10	24	4	105	17	9	21.52	28.74	0.379	0.621
37	5	92	9	8	9	29	22	25.45	28.69	0.828	0.172
38	5	37	3	19	5	83	22	21.56	30.61	0.509	0.491
39	1	132	12	7	5	4	9	21.05	27.39	0.219	0.781
40	4	5	8	28	75	34	19	22.87	29.93	0.556	0.444
41	2	13	83	33	14	5	21	23.48	28.63	0.509	0.491
42	3	8	25	45	7	59	25	24.09	31.29	0.734	0.266
43	2	15	36	11	40	42	25	24.38	32.03	0.787	0.213
44	2	7	127	5	4	9	17	19.71	28.09	0.249	0.751
45	5	17	9	11	28	83	21	22.37	29.77	0.509	0.491
46	3	11	26	65	19	23	25	23.71	29.68	0.615	0.385
47	5	10	9	7	2	118	23	20.00	28.60	0.302	0.698
48	3	19	22	33	13	31	51	25.10	29.76	0.805	0.195
49	2	20	38	25	24	22	40	24.11	32.53	0.775	0.225
50	5	9	28	58	10	34	30	25.82	26.74	0.789	0.201
51	4	22	7	29	16	51	44	25.78	28.19	0.905	0.095
52	4	70	11	25	13	22	28	25.44	32.77	0.923	0.077
53	1	31	33	17	18	32	38	24.33	33.48	0.817	0.183
54	1	58	23	13	45	5	25	24.57	28.76	0.657	0.343
55	4	16	52	11	14	27	49	25.72	29.21	0.917	0.083
56	1	63	27	4	11	37	27	22.81	31.38	0.627	0.373
57	3	4	3	132	1	1	28	21.54	27.26	0.219	0.781
58	1	79	11	12	8	9	50	22.82	29.63	0.533	0.467
59	2	19	90	14	1	8	37	22.54	29.04	0.467	0.533
60	4	57	17	11	40	2	42	24.65	30.38	0.763	0.237
61	1	43	6	10	15	49	46	24.38	30.77	0.746	254
62	2	52	62	3	5	3	44	25.13	27.52	0.633	0.367
63	1	30	20	13	11	39	56	24.57	32.67	0.822	0.178
64	2	11	34	17	17	29	61	25.01	29.94	0.799	0.301
65	4	7	8	5	84	12	53	22.91	29.14	0.503	0.497
66	2	14	41	14	5	33	62	25.22	28.46	0.757	0.243
67	1	61	7	14	12	21	54	24.50	28.67	0.639	0.361

TABLE VI (continued)

Item	Key	Alternatives a	b	c	d	e	Omit	Q-Mean	P-Mean	Q-Ratio	P-Ratio
68	3	12	9	62	4	26	56	23.87	29.69	0.633	0.367
69	3	1	4	90	15	5	54	23.85	27.90	0.467	0.533
70	4	1	13	1	16	0	138	24.84	37.13	0.905	0.095
71	3	15	30	63	1	2	58	22.14	32.51	0.627	0.373

Test items whose P-ratio was in the interval from 0.200 to 0.800 were accepted for further consideration. Preference was given to those items that were close to 0.500. If the mean of those who passed the item exceeded the mean of those who failed the item and all alternatives were chosen, the item was selected for possible inclusion in the final form of the test without modification. In order to obtain two items for each competency, it was necessary to use some items whose P-ratio's were less than 0.200. These items were modified to correct what appeared to be their major weakness. Finally, six new items were written since equivalent items in the original pool were not acceptable. In writing the new items, the author made every effort to stay within the guidelines suggested by the panel of judges.

The completed form of the achievement test may be found in Appendix D. Table VII lists the distribution of responses, the Q-mean, the P-mean, the Q-ratio, and the P-ratio for the 517 students who completed the final form of the achievement test. The reliability of the final form of the achievement test, as calculated

by the Kuder Richardson Formula 20,

$$r_{xx} = \frac{n}{n-1} \frac{S_x^2 - \Sigma pq}{S_x^2}$$

is 0.86.

TABLE VII

ITEM ANALYSIS OF BEAL'S MATHEMATICAL COMPETENCY TEST

Item	Key	Alternatives a	b	c	d	e	Omit	Q-Mean	P-Mean	Q-Ratio	P-Ratio
1	2	83	163	39	115	94	23	20.69	26.48	0.685	0.315
2	5	22	114	9	12	356	4	19.66	23.81	0.311	0.689
3	3	34	16	347	17	90	13	18.35	24.56	0.329	0.671
4	5	57	57	33	24	333	13	18.42	24.79	0.356	0.644
5	3	107	59	158	114	56	23	20.74	26.57	0.694	0.306
6	3	9	55	356	25	56	16	16.42	25.28	0.311	0.689
7	2	11	244	50	122	78	12	19.47	25.93	0.528	0.472
8	4	270	59	26	115	34	13	21.48	26.15	0.778	0.222
9	1	333	64	66	15	26	13	18.07	24.98	0.356	0.644
10	2	197	127	62	42	67	22	21.96	24.24	0.754	0.246
11	2	153	267	34	12	33	18	20.92	24.02	0.484	0.516
12	1	290	46	57	38	64	22	18.30	25.83	0.439	0.561
13	4	26	51	64	182	182	12	19.36	28.34	0.648	0.352
14	1	308	56	42	17	82	12	17.27	26.08	0.404	0.596
15	3	101	94	155	54	91	22	21.06	25.94	0.700	0.300
16	5	148	35	18	60	235	21	19.09	26.63	0.545	0.455
17	4	36	20	65	364	17	15	16.44	25.08	0.296	0.704
18	1	138	63	182	40	80	14	20.28	28.68	0.733	0.267
19	1	122	78	99	47	147	24	21.17	26.90	0.764	0.236
20	3	135	16	230	34	87	15	19.03	26.87	0.555	0.445
21	4	30	65	35	307	59	21	17.58	25.90	0.406	0.594
22	4	25	21	57	326	66	22	18.25	25.02	0.369	0.631
23	1	290	39	59	49	63	17	17.93	26.11	0.439	0.561
24	3	44	87	166	34	160	26	20.44	26.93	0.679	0.321
25	3	47	75	165	93	116	21	20.13	27.61	0.681	0.319
26	3	37	79	114	11	244	32	21.01	27,84	0.779	0.221
27	1	374	36	34	25	28	20	14.94	25.42	0.277	0.723
28	4	40	29	75	208	134	31	19.14	27.54	0.598	0.402

TABLE VII (continued)

Item	Key	Alternatives a	b	c	d	e	Omit	Q-Mean	P-Mean	Q-Ratio	P-Ratio
29	1	166	67	60	24	171	29	20.44	26.92	0.679	0.321
30	1	247	177	35	22	9	27	20.14	25.12	0.522	0.478
31	2	291	67	25	25	85	24	22.51	22.61	0.870	0.130
32	3	44	50	202	63	115	43	20.38	25.86	0.609	0.391
33	1	105	146	67	50	100	49	21.67	25.86	0.797	0.203
34	5	45	46	39	33	319	35	16.87	26.03	0.383	0.617
35	2	276	106	37	28	33	37	21.96	24.71	0.795	0.205
36	3	68	53	223	33	103	37	18.77	27.47	0.569	0.431
37	1	152	56	180	34	46	49	20.85	26.53	0.706	0.294
38	2	181	87	53	42	99	55	21.70	26.59	0.832	0.168
39	2	43	258	52	57	58	49	18.47	26.58	0.501	0.499
40	4	40	160	41	174	52	50	20.41	26.68	0.663	0.337
41	2	15	344	19	28	46	65	16.84	25.38	0.335	0.665
42	2	48	192	113	67	33	64	19.54	27.56	0.629	0.371
43	2	75	95	50	82	141	74	21.90	25.29	0.816	0.184
44	1	171	71	52	52	97	74	20.36	26.89	0.669	0.331
45	2	38	169	46	60	127	77	19.91	27.89	0.673	0.327
46	5	74	30	39	64	241	69	18.73	26.86	0.534	0.466
47	3	70	32	232	18	90	75	18.71	27.21	0.551	0.449
48	4	49	44	94	186	64	80	19.66	27.60	0.640	0.360
49	3	28	23	287	61	43	75	17.67	26.41	0.445	0.555
50	5	19	26	33	23	339	77	15.84	26.03	0.344	0.656
51	5	38	47	43	21	281	87	17.56	26.69	0.456	0.544
52	3	79	87	227	15	18	91	18.28	27.94	0.561	0.439

Table VIII provides the reader with a listing of the competencies indexed by test items found on Beal's Mathematics Competency Test.

SELECTION OF ATTITUDE SCALE

The Revised Math Attitude Scale (Appendix E) was developed by Lewis R. Aiken and appeared in Scales for the Measurement of Attitudes by Shaw and Wright. The test consisted of ten positive

TABLE VIII

COMPETENCY INDEXED BY TEST ITEM

Competency	Test Items
1	7,42
2	13,26
3	27,46
4	2,51
5	17,36
6	39,50
7	14,25
8	3,34
9	15,30
10	9,40
11	37,48
12	22,45
13	4,19
14	5,32
15	1,23
16	38,41
17	24,49
18	18,23
19	6,33
20	10,35
21	43,44
22	47,20
23	12,21
24	8,31
25	16,29
26	11,52

and ten negative items which required responses on a five-point Likert Scale. Scoring was accomplished by awarding points one through five. The following illustrates the basis for scoring the test:

	Strongly Disagree	Disagree	Undecided	Agree	Strongly Agree
Positive items	1	2	3	4	5
Negative items	5	4	3	2	1

The maximum score attainable was 100 and the minimum score 20. The test has a reliability coefficient of 0.94.

Pretest

All students participating in the study completed The Revised Math Attitude Scale as a pretest in September, 1971. In one control class, it was necessary for the teacher to read the attitude statements. The student indicated that he understood what the statements meant and circled the responses as the teacher read the items.

Posttest

The Revised Math Attitude Scale and Beal's Mathematics Competency Test were given as posttests to all students at the end of the first semester. In most cases this was at the end of January. Both tests were mailed to each teacher and administered to experimental and control classes.

Visitations to Schools

The author visited six of the participating schools during February in order to personally interview students, teachers, and administrators to determine their attitudes toward the use of the conference materials. These interviews were recorded on audio-tape at the time of the visits and later transferred to written manuscript. The interviews were not structured, but rather were left open to the comments that the individual wanted to make relative to the effectiveness of the materials. The results of these interviews are treated in detail in Chapter IV.

SUMMARY

During the summer of 1971, participants in an NSF sponsored Conference for Teachers of Low Achievers in Mathematics developed materials to help the low achiever attain twenty-six basic mathematical competencies. In order to provide an evaluation of these materials, thirteen teachers from ten different schools were selected to use the activities to supplement their regular materials in an experimental class while using only the regular materials in a control class during the fall semester of the 1971-72 school year. Since one teacher agreed to teach three experimental/control classes, a total of fifteen different pairs of classes were involved in the study.

The author constructed a 52-item multiple choice test to measure the 26 competencies. This test was given as a posttest to all students. Also, an attitude test was administered as both a pretest and posttest. Finally six of the schools were visited by the writer where

he obtained verbal comments relative to the effectiveness of the materials from students, teachers, and administrators.

CHAPTER IV

FINDINGS

It was the purpose of this study to investigate the effects of activity oriented materials, developed to help the low achiever attain basic mathematical competencies, on student attitude and achievement in mathematics.

The null hypotheses to be tested were: (1) There are no significant differences between the attitude toward mathematics of the experimental classes and the attitude toward mathematics of the control classes. (2) There are no significant changes in attitude toward mathematics by the experimental classes. (3) There are no significant changes in attitude toward mathematics by the control classes. (4) There are no significant differences between the achievement of the experimental classes and the achievement of the control classes.

This chapter is divided into four major sections: results of achievement testing; results of attitude testing; results of unit evaluations; and results of school visitations. To facilitate the reporting of data within this and subsequent chapters the experimental and control classes in different schools have been identified by the symbols shown in Table IX.

TABLE IX

SYMBOLS USED TO IDENTIFY DIFFERENT SCHOOLS

School	Experimental Class	Control Class
Westlane Junior High	1 A	1 B
Bishop Hartley High	2 A	2 B
New Hartford Junior High	3 A	3 B
New Hartford Junior High	4 A	4 B
New Hartford Junior High	5 A	5 B
Maple Heights West Junior High	6 A	6 B
Prince Edward Academy	7 A	7 B
Elkhorn High	8 A	8 B
Marian High	9 A	9 B
Lincoln Junior High	10 A	10 B
Technical High	11 A	11 B
Carl Sandburg Middle School	12 A	12 B
Carl Sandburg Middle School	13 A	13 B
Carl Sandburg Middle School	14 A	14 B
Carl Sandburg Middle School	15 A	15 B

RESULTS OF ACHIEVEMENT TEST

Beal's Mathematics Competency Test was constructed to measure the twenty-six competencies for which conference materials were developed. This test was given as a posttest to all experimental

and control classes. A computer program was written to select any combination of test items, score the items, and then punch this subscore into another computer card. This program allowed the author to evaluate achievement using only those test questions that measured the competencies for which the teacher had selected conference materials. Table X provides the reader with a listing of the competencies for which conference materials were selected in each of the experimental classes.

See Appendix F for tables that contain percentage distributions of responses, by competency, for each of the classes involved in the study.

Since each teacher was allowed to select the competencies for which they would use conference materials, the author chose to use t-ratios to test for significant differences between means of experimental and control classes. The experimental and control classes could be considered independent samples. Therefore, the writer selected the formula[1] used in calculating t-ratios.

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{(n_1-1)\ s_1^2 + (n_2-1)\ s_2^2}{n_1 + n_2 - 2}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

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[1]Gene V. Glass, and Julian C. Stanley, Statistical Methods in Education and Psychology (Englewood Cliffs, New Jersey, 1970), p. 295

TABLE X

COMPETENCIES SELECTED FOR USE
BY EXPERIMENTAL CLASSES

Experimental Class	Competencies Selected
1 A	1, 2, 3, 4, 5, 6
2 A	4, 7, 13, 16, 17, 20, 21, 22, 23
3 A	2, 3, 4, 5, 6, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 21, 22, 23, 24, 25
4 A	2, 3, 4, 5, 6, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 21, 22, 23, 24, 25
5 A	2, 3, 4, 5, 6, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 21, 22, 23, 24, 25
6 A	1, 2, 3, 4, 5, 6
7 A	1, 2, 3, 4, 10, 11
8 A	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 16, 17
9 A	1, 2, 4, 5, 18, 20
10 A	2, 3, 4, 5, 6, 10, 11, 12, 13, 14, 15, 17, 18, 19
11 A	1, 2, 3, 4, 5, 6
12 A	1, 2, 3, 4, 5, 6
13 A	1, 2, 3, 4, 5, 6
14 A	1, 2, 3, 4, 5, 6
15 A	1, 2, 3, 4, 5, 6

Table XI is a summary of the t-ratios calculated for each of the experimental and control classes.

TABLE XI

SUMMARY OF t-RATIOS BEAL'S MATHEMATICS COMPETENCY TEST

Class	Number of Test Items	Mean	Variance	df	t-Ratio
1 A	12	3.550	4.471	35	-0.673
1 B		4.059	6.184		
2 A	18	10.125	5.274	63	2.469*
2 B		8.606	6.996		
3 A	42	14.600	18.044	22	-1.131
3 B		16.357	11.324		
4 A	42	22.889	46.928	29	-0.346
4 B		23.692	31.897		
5 A	42	19.750	41.250	32	-1.086
5 B		22.071	32.379		
6 A	12	5.704	9.909	51	3.461**
6 B		3.192	3.922		
7 A	12	8.222	2.536	34	1.614
7 B		7.278	3.624		
8 A	32	21.000	2.800	22	-0.728
8 B		22.167	14.147		
9 A	12	3.765	4.441	26	0.051
9 B		3.727	2.418		
10 A	28	19.000	5.636	52	3.775**
10 B		14.935	22.396		
11 A	12	4.632	5.801	24	-0.237
11 B		4.857	1.143		

TABLE XI (continued)

Class	Number of Test Items	Mean	Variance	df	t-Ratio
12 A	12	6.333	10.524	27	o.859
12 B		5.357	7.632		
13 A	12	3.846	5.308	25	-0.264
13 B		4.071	4.533		
14 A	12	4.167	8.697	22	-0.077
14 B		4.250	5.295		
15 A	12	6.000	3.500	23	1.377
15 B		7.250	6.932		

*Significant $p < 0.05$
**Significant $p < 0.01$

Seven of the experimental classes had means greater than their corresponding control classes, and of these seven, three had significantly greater means. In no case was the control class mean significantly greater than the experimental class mean. As a result of the statistical tests, the hypothesis that there are no significant differences between the achievements of the experimental classes and the control classes is rejected for three of the classes and is not rejected for the remaining twelve.

RESULTS OF THE ATTITUDE TEST

The Math Attitude Scale was administered as a pretest and as a posttest. This allowed the author to determine if there were any significant changes during the semester in attitude towards mathematics as well as checking for differences in attitude between experimental and control classes.

According to Ferguson, if the data consists of paired measurements on the same group of individuals, a procedure different from that for independent samples should be used. Therefore, the author selected the difference method for calculating the t-ratios between pretest and posttest mean scores. The formula[2]

$$t = \frac{\Sigma D}{\sqrt{(N\Sigma D^2 - (\Sigma D)^2)/\ (N - 1)}}$$

where D is the difference between pretest and posttest scores and N is the number of subjects, was used in calculating the t-ratios by the difference method.

Since each experimental class was allowed to use different activities, the writer chose to analyze the attitude scores for each of the classes separately. Table XII is a summary of t-ratios for

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[2]George A. Ferguson, Statistical Analysis in Psychology and Education (2nd ed.; New York: McGraw-Hill Book Company), p. 170.

differences between pretest and posttest attitude scores for the experimental classes. The difference mean listed for each class is the average of all differences between pretest and posttest attitude scores.

TABLE XII

SUMMARY OF t-RATIOS DIFFERENCES BETWEEN PRETEST AND POSTTEST SCORES OF EXPERIMENTAL CLASSES REVISED MATH ATTITUDE SCALE

Class	Difference Mean	Variance	S.D.	df	t-Ratio
1 A	7.800	180.599	13.439	14	2.248*
2 A	3.406	81.733	9.041	31	2.131*
3 A	6.571	507.285	22.523	6	0.772
4 A	3.000	370.400	19.246	10	0.517
5 A	5.583	106.447	10.318	11	1.875
6 A	6.375	198.418	14.086	23	2.217*
7 A	0.500	249.500	15.796	13	0.118
8 A	4.000	81.600	9.033	5	1.085
9 A	3.688	235.962	15.361	15	0.960
10 A	0.240	157.773	12.561	24	0.096
11 A	4.444	118.732	10.896	17	1.730
12 A	3.667	273.952	16.552	14	0.858
13 A	-4.154	175.641	13.253	12	-1.130
14 A	8.400	94.933	9.743	9	2.726*
15 A	-6.267	186.924	13.672	14	-1.775

*Significant $p < 0.05$

Thirteen of the fifteen experimental classes exhibited a more positive attitude towards mathematics after having used the conference materials for one semester. Four of the experimental classes had significantly higher means, at the 0.05 level, on the posttest than on the pretest. Therefore, the hypothesis that there are no significant changes in attitude toward mathematics by the experimental classes is rejected for four of the classes and is not rejected for eleven of the classes.

Table XIII provides the reader with a summary of t-ratios for differences between pretest and posttest attitude scores for the control classes. Again, the difference mean is the average of all differences between pretest and posttest attitude scores. Seven of the control classes exhibited a more positive attitude toward mathematics after one semester and of the seven classes, two had significantly better attitude scores at the 0.05 level and one had significantly better attitude scores at the 0.01 level.

Hence, the hypothesis that there are no significant changes in attitude toward mathematics of the control classes is rejected for three of the classes and not rejected for twelve of the classes.

TABLE XIII

SUMMARY OF t-RATIOS DIFFERENCES BETWEEN PRETEST AND POSTTEST SCORES OF CONTROL CLASSES REVISED MATH ATTITUDE SCALE

Class	Difference Mean	Variance	S.D.	df	t-Ratio
1 B	-3.214	127.566	11.295	13	-1.065
2 B	5.636	205.863	14.348	32	2.257*
3 B	9.714	438.904	20.950	6	1.227
4 B	0.000	290.000	17.029	5	0.000
5 B	13.000	151.090	12.292	11	3.664**
6 B	-0.840	326.973	18.082	24	-0.232
7 B	-4.059	421.058	20.520	16	-0.816
8 B	7.000	134.875	11.614	16	2.485*
9 B	-1.273	200.218	14.150	10	-0.298
10 B	1.260	94.383	9.715	22	0.622
11 B	7.200	33.700	5.805	4	2.773
12 B	-2.571	83.802	9.154	13	-1.051
13 B	-8.083	147.356	12.139	11	-2.307
14 B	9.091	211.091	14.528	10	2.075
15 B	-1.833	107.606	10.373	11	-0.612

*Significant $p < 0.05$

**Significant $p < 0.01$

Table XIV is a summary of t-ratios for differences between experimental and control class posttest means.

TABLE XIV

SUMMARY OF t-RATIOS POSTTEST MEANS
THE REVISED MATH ATTITUDE SCALE

Class	Mean	Variance	S.D.	df	t-Ratio
1 A	59.267	129.781	11.392	27	-0.733
1 B	62.500	153.192	12.377		
2 A	59.844	219.297	14.809	63	1.507
2 B	54.849	290.256	19.755		
3 A	61.714	129.905	11.398	12	-1.636
3 B	70.000	49.670	7.047		
4 A	66.909	311.691	17.655	15	0.790
4 B	60.667	103.067	10.152		
5 A	63.333	221.151	14.871	22	-1.649
5 B	71.750	91.477	9.564		
6 A	72.583	209.557	14.476	47	2.377*
6 B	61.200	349.332	18.690		
7 A	48.429	386.263	19.654	29	-0.347
7 B	50.882	382.234	19.551		
8 A	73.000	70.000	8.367	21	1.742
8 B	63.765	147.066	12.127		
9 A	65.750	313.933	17.718	25	0.466
9 B	62.909	135.091	11.623		
10 A	62.760	113.523	10.655	46	-0.407
10 B	64.435	300.983	17.349		
11 A	62.500	107.559	10.371	21	-0.343
11 B	64.200	46.700	6.834		
12 A	63.800	288.457	16.984	27	0.533
12 B	60.714	194.220	13.936		

TABLE XIV (continued)

Class	Mean	Variance	S.D.	df	t-Ratio
13 A	66.462	263.436	16.231	23	0.921
13 B	60.333	290.242	17.037		
14 A	87.500	63.389	7.962	19	2.583**
14 B	73.545	233.273	15.273		
15 A	46.800	368.171	19.188	25	-0.979
15 B	54.167	389.424	19.734		

*Significant $p < 0.05$

**Significant $p < 0.01$

As stated in the section on achievement testing, the writer selected the formula

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{(n_1 - 1)\, s_1^2 + (n_2 - 2)\, s_2^2}{n_1 + n_2 - 2}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

to be used in calculating the t-ratios for Table XIV since the experimental and control classes could be considered to be independent samples.

Eight of the fifteen experimental classes had higher posttest means than their corresponding control classes. One of the eight was significantly higher at the 0.05 level and one at the 0.01 level. Therefore, the hypothesis that there are no significant differences between the attitude toward mathematics of the experimental classes

and the attitude toward mathematics of the control classes is rejected for two of the classes and is not rejected for the remaining thirteen classes.

RESULTS OF UNIT EVALUATIONS

Each teacher was asked to complete a unit summary (Appendix G) for each unit used in the experimental class. These results are summarized in this section.

It was interesting to note that in all cases, the teachers listed all students in the experimental classes as having used the conference materials. It was the intent of the author that the materials would be used only by those students who would profit by more activities. The amount of time devoted to any particular unit ranged from one to sixteen days with the average number of days being 3.6. In all but three cases reported, the conference materials were used in conjunction with the textbook and were not used to replace it. The amount of class time devoted to the conference materials ranged from 10 per cent to 100 per cent with the average being 35 per cent. This does not mean that 35 per cent of the total class time during the semester was devoted to the study of conference materials, but rather that 35 per cent of the time was devoted to developing the twenty-six competencies for which conference materials were written.

In addition to collecting the previously mentioned data, the teachers were asked to list suggestions they might have for improving the materials. Many suggestions involved legibility, punctuation,

and organization; however, some comments were made that applied to the units in general. Below are some of those that occurred with the greatest frequency:

1. More activities should be included for operations on whole numbers, fractions, and decimal fractions.
2. The teacher needs to be able to remove the activity sheets and duplicate them for the students.
3. Students were reluctant to use activities that involved manipulative materials easily identifiable as having been used in the elementary grades.
4. When using the units involving measurement, more time should be devoted to learning how to use the ruler.

RESULTS OF SCHOOL VISITATIONS

In order that the author could obtain more feedback relative to the effectiveness of the materials, the author selected six schools to visit personally. During these visits, reactions to the conference materials were obtained from teachers and students of experimental classes and from school administrators. Their comments were recorded on audio tape and later transferred to written manuscript. Some of the comments that occurred with greatest frequency are listed below under the categories of students, teachers, and administrators.

Reactions of Students

1. I like to do things not in the book. The book is work.

2. This class is not like the others, we don't have to do work. We play games.
3. It makes what we are doing in the book easier.
4. We don't have to work all the time.
5. It's fun.
6. It's different.
7. It's O.K.

Reactions of Teachers

1. When the material doesn't come from the book, it is not viewed as work.
2. Materials seem to work best as supplementary and should not be used every day.
3. The instructions have not always been clear.
4. The reading level of the directions to the student is too high in some of the units.
5. It helps to have a choice of activities.
6. The units helped to open a new view of routine work to most of my class.
7. It was not possible to give all the students the work unassisted.
8. I found the competencies helpful in selecting the materials to be used in class.
9. The activities should be on separate pages so that ditto masters may be made directly from the materials.

10. We need more activities for operations with whole numbers, fractions and decimals.

Reactions of Administrators

1. Activities should not be keyed to too many commercial games because of the expense involved.
2. Sometimes it is difficult to get parents to realize that all students in the same grade do not have to use the same materials.
3. Teachers seemed to be excited about using the materials.
4. Any materials that make the work of my teachers more effective are appreciated.
5. One parent visited school the day his child was involved in an activity that utilized a commercial game. He commented that he wished he could have played games in math.

SUMMARY OF THE FINDINGS

It was the purpose of this study to evaluate the effects of activity oriented materials, developed to help the low achiever attain basic mathematical competencies, on student attitude and achievement in mathematics.

Beal's Mathematical Competency Test was given after the students in the experimental classes had used various combinations of conference materials for one year. Since each class used a unique combination of activities, differences between experimental and control

classes were checked for significance by t-ratios. Seven of the fifteen experimental classes had means greater than their corresponding control classes and of these seven classes, three had significantly greater means.

The Revised Math Attitude Scale was administered to all students as a pretest and posttest. Again, individual differences were analyzed by calculating their t-ratios. Thirteen of the fifteen experimental classes exhibited a more positive attitude towards mathematics after having used the conference materials for one semester, and of the thirteen, four had differences significant at the 0.05 level. When experimental classes were compared to their corresponding control classes on the posttest, eight of the fifteen experimental classes had higher mean scores than their corresponding control classes. Of the eight with higher mean scores, one difference was significant at the 0.05 level and one at the 0.01 level.

Students, teachers, and administrators were enthusiastic in their support for the conference materials. The teachers felt the materials provided their students a new and refreshing method for developing basic mathematical competencies.

CHAPTER V

SUMMARY AND CONCLUSIONS

The first purpose of this study was to evaluate the effectiveness of the materials developed in the National Science Foundation Conference, based on active involvement in the learning process by the learner, in helping the low achiever attain basic mathematical competencies. The second purpose of this study was to determine if the low achiever has a more positive attitude towards mathematics after using the materials.

The null hypotheses to be tested were: (1) There are no significant differences between the attitude toward mathematics of the experimental classes and the attitude toward mathematics of the control classes. (2) There are no significant changes in attitude toward mathematics by the experimental classes. (3) There are no significant changes in attitude toward mathematics by the control classes. (4) There are no significant differences between the achievement of the experimental classes and the achievement of the control classes.

PROCEDURE

This study involved thirteen teachers in fifteen classes located in six different states. Ten of the teachers were selected from twenty-five participants in a National Science Foundation Summer

Conference for Secondary Teachers of Low Achievers in Mathematics held at the University of Nebraska in the summer of 1971. The other three teachers were from one of the schools represented by a conference member who agreed to participate in the study. During the first semester of the 1971-1972 school year each teacher agreed to teach at least one experimental and one control class. Since one teacher agreed to teach three experimental and three control classes, there were fifteen experimental and fifteen control classes of low achievers involved in the study.

The members of the NSF Conference developed materials to be used in helping the low achiever attain twenty-six basic mathematical competencies selected from those proposed by the Committee on Basic Mathematical Competencies of the National Council of Teachers of Mathematics.

Teachers of an experimental class agreed to use conference materials to supplement their regular curriculum in helping their students attain at least six of the twenty-six competencies. The control classes used the regular curriculum materials while the experimental classes used the conference materials. The control classes, in most instances, studied the same "regular" materials as the experimental classes, only they studied them for a longer period of time.

In order to test for attitude change, The Revised Math Attitude Scale was administered to all classes as a pretest in early September and as a posttest in late January. Beal's Mathematics Competency

Test was constructed by the author to measure the achievement of the twenty-six competencies for which materials were developed by the NSF conference members. This test was administered as a posttest in late January. In addition to collecting data with the two tests, teachers completed unit evaluations for each unit of conference materials used in the experimental classes. Finally, six of the ten schools were visited, where the writer interviewed administrators, teachers, and student who were involved in the study.

Since each teacher was allowed to select any combination of units from the conference materials to use in the experimental class, it was necessary for the author to evaluate experimental classes separately to see if the selected units helped the students attain the competencies for which the units were written. This was accomplished by the author writing a computer program to calculate a subscore for any combination of items from Beal's Mathematics Competency Test. Differences between mean subscores for experimental and control classes were then tested for significance by calculating t-ratios. Similarly, t-ratios were calculated for differences between mean attitude scores of experimental and control classes.

FINDINGS

Statistical techniques consisting of the t-test for independent samples and the "difference" t-test for repeated measures on the same sample resulted in the following findings:

1. During the duration of the study, there was a statistically

significant increase in favorable attitude toward mathematics in four of the fifteen experimental classes as measured by The Revised Math Attitude Scale. The level of significance was 0.05. There was no statistically significant decrease in favorable attitude toward mathematics in any of the experimental classes.

2. During the duration of the study, there was a statistically significant increase in favorable attitude toward mathematics in three of the fifteen control classes as measured by The Revised Math Attitude Scale. The level of significance for two of the classes was 0.05 and for one of the classes was 0.01. There was no statistically significant decrease in favorable attitude toward mathematics in any of the control classes.

3. At the termination of the course, students in two of the fifteen experimental classes had a statistically significant more favorable attitude toward mathematics than their corresponding control classes as measured by The Revised Math Attitude Scale. The level of significance for one class was 0.01 and for one class 0.05. In no control class was there a statistically significant more favorable attitude toward mathematics than their corresponding experimental class.

4. Student achievement in three of the fifteen experimental classes was significantly better than their corresponding control classes as measured by Beal's Mathematics Competency Test. The level of significance for one difference was 0.05 and for two differences was 0.01. No control class had a significantly better student

achievement mean than its corresponding experimental class mean.

Teachers, who used the conference materials, indicated in most instances that the conference materials were used to supplement rather than replace regular materials. Also, for most units the teachers used the materials with the entire class rather than with individual students. There seemed to be a consensus of opinion that more activities were needed for basic operations with whole numbers, fractions and decimal fractions. Finally, teachers, administrators, and students who participated in the study were enthusiastic in their support for the conference materials and believed the materials were not only "fun," but also made the work in the regular textbook easier.

CONCLUSIONS

The results obtained from The Revised Math Attitude Scale led the writer to conclude that even though there was no significant decrease in attitude by classes using the conference materials, there was not sufficient evidence to support the conclusion that there was significant increase. However, it should be noted that verbal comments from students, teachers, and administrators would support the contention that the students enjoyed using the conference materials more than regular materials.

The results of administering Beal's Mathematics Competency Test do not support the conclusion that the use of conference materials will result in significant differences in achievement from that obtained by using regular class materials. However, again, the verbal

reactions of students, teachers, and administrators would lead one to believe that the students enjoyed studying the conference materials more than the regular materials.

RECOMMENDATIONS

As a result of conducting an evaluation of activity-oriented materials developed to help the low achiever attain basic mathematical competencies, the author makes the following recommendations:

1. Schools should consider providing supplementary materials to the textbook for the low achiever.
   (a) The supplementary materials selected should provide for active involvement on the part of the learner.
   (b) The supplementary materials should contain a sufficient number of activities to allow the student a choice.
   (c) The supplementary materials should require little time on the part of the teacher for selection and preparation.
2. New curriculum materials in mathematics should be evaluated to determine the effects of the materials on student attitude and achievement in mathematics.
3. The NSF materials, developed in the summer of 1971, should be revised to reflect the recommendations of the students, teachers and administrators involved in this study.

4. The NSF materials should be expanded to include more competencies.

SUGGESTIONS FOR FURTHER RESEARCH

1. Research is needed to determine whether low achievers, classified by a variety of variables, would profit by the use of the NSF materials.

2. Research is needed to determine whether the low achiever has a more favorable attitude toward teacher-prepared or commercially-prepared materials.

3. Research is needed to determine the value of "choice in materials" allowed the low achiever in improving his attitude towards mathematics.

4. Research is needed to determine the effects of materials written at various reading levels on attitude and achievement of low achievers in mathematics.

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APPENDIX A

Sample Units of Instruction

COMPETENCY: Write an equivalent fraction for a given fraction.

OBJECTIVES:

1. Given a fraction not in lowest terms, the student will be able to write the equivalent fraction in lowest terms.
2. Given a fraction the student will be able to write at least one equivalent fraction.

ACTIVITY 1:

Suggested Materials:

Nine pie pans and nine different color disks to fit in the pans. Divide the pie pans into fractional parts (halves, 3rds, 4ths, 5ths, 6ths, 8ths, 9ths, 10ths and 12ths). Label each 2/2, 3/3, 4/4, etc. Cut disks into corresponding fractional parts (1/2's, 1/3's, 1/4's, etc.). Make enough sets so that every three or four students have a set. Directions to the student should be on a worksheet on which he may write.

Directions to the student:

Place the pie pans with the color wedges in front of you and examine them. Compare the sizes of different colored wedges.

1. How many 1/4 wedges does it take to make a 1/2 wedge?

   _____

2. How many 1/8 wedges does it take to make 1/2 wedge?

   _____

3. How many 1/6's does it take to make 1/2?

   _____

We can write:
two "1/4's" as 2/4;
four "1/8's" as 4/8
three "1/6'a" as 3/6;
Each of these is equivalent to 1/2.
2/4 = 1/2
4/8 = 1/2
3/6 = 1/2

1/2, 2/4, 4/8, 3/6 are equivalent fractions. They are different names for the same fraction. Look again at the pie pans and colored wedges.

4. How many "1/9's" make 1/3? _____

5. 1/3 = ____/9.

6. How many "1/6's" does it take to make 1/3?
   _____

7. ____/9 = 1/3 and ____/6 = 1/3.

8. ____/9, ____/6, and 1/3 are equivalent fractions.

Use the pie pans and wedges to fill in the missing numerators or denominators.

9. ____/4 = 1/2
10. ____/12 = 1/2
11. ____/12 = 1/4
12. ____/8 = 1/4
13. ____/10 = 1/5
14. 2/____ = 1/5
15. 3/____ = 1/4
16. 2/____ = 1/3

If you have trouble with exercises 17 or 18, use the answers to problems 9 through 16 to help you.

17. Name two fractions that are equivalent to 1/4 =
    _____ = _____
18. Name two fractions that are equivalent to 1/5 =
    _____ = _____

Suggested strategies:

1. The student should be familiar with the parts of a fraction such as numerator and denominator.
2. The teacher may want to go over worksheets with the students.
3. The worksheets may be done in a small group.
4. Working with the teacher might be best if the students have not practiced working alone.
5. The teacher may supplement the worksheet with more problems on worksheets or from texts.

ACTIVITY 2:

Suggested materials:

Same as for Activity 1.

Directions to the student:

Using the pie pans and colored wedges please answer the following questions:

1. Take 2/5 (two 1/5 wedges put together). How many 1/10's does it take to make 2/5's?

   _____

2. How many 1/12's does it take to make 3/4's (three 1/4 wedges put together)?

   _____

Use the pie plates and colored wedges to fill in the missing numerators and denominators.

3. 5/6 = ____/12
4. 2/3 = 4/____
5. 2/3 = ____/12
6. 4/5 = ____/10
7. 4/8 = 1/____
8. 4/6 = ____/3
9. 3/4 = 6/____
10. 3/5 = 6/____
11. Name two equivalent fractions for 2/3 = _____ = _____

    Use exercises 1 through 10 to help you answer questions 11 and 12.

12. Name two equivalent fractions for 3/4 = _____ = _____

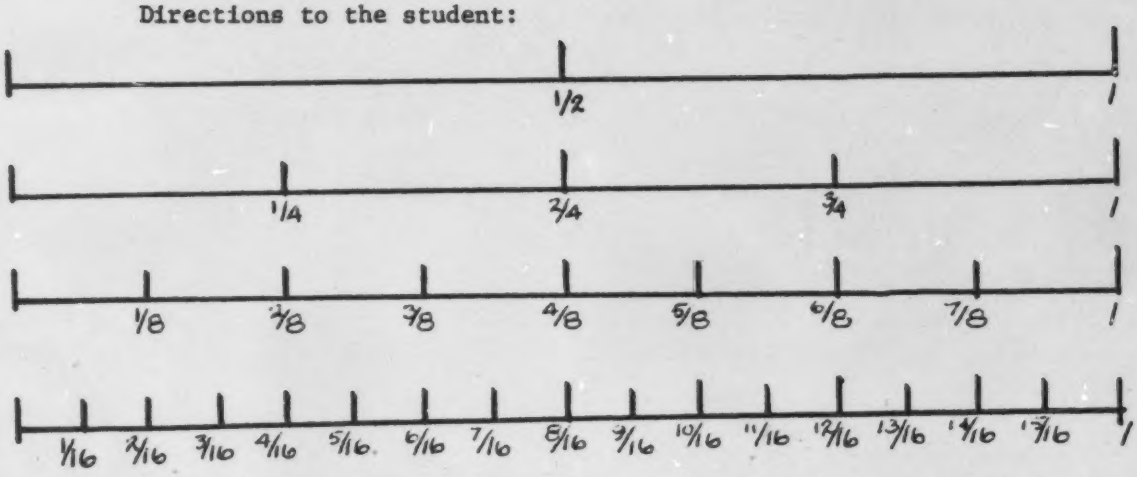
Suggested strategies:

If you did activity 1 with the students, you might try letting them do this activity in small groups of three or four.

ACTIVITY 3:

Suggested materials:
worksheet

Directions to the student:



Use the drawing above to help you decide which of the following fractions are equivalent. Place an equal sign (=) between each pair of equivalent fractions. (Do nothing if the pair is not equivalent.

1. 1/4 ____ 2/8
2. 3/4 ____ 12/16
3. 14/16 ____ 7/8
4. 1/2 ____ 3/8
5. 7/8 ____ 10/16
6. 1/2 ____ 8/16
7. 5/8 ____ 10/16
8. 6/16 ____ 3/8
9. 4/16 ____ 2/8
10. 6/8 ____ 12/16
11. 6/8 ____ 3/4
12. 4/8 ____ 6/16

Circle the fractions that are in lowest terms.

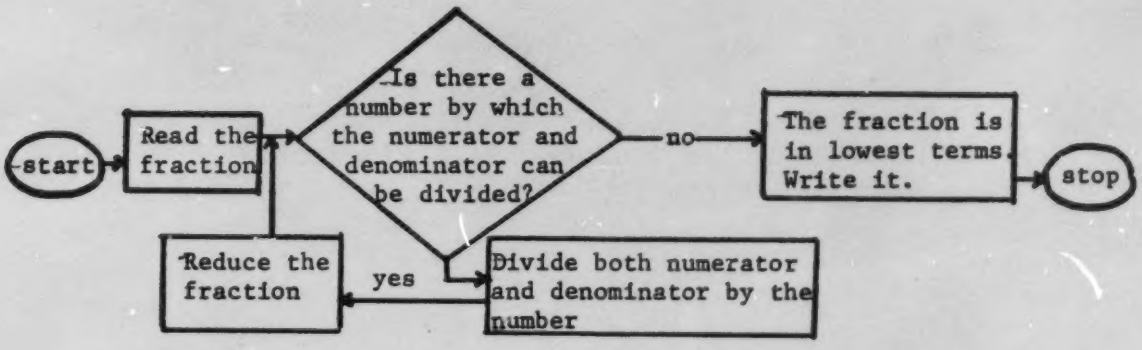
Suggested strategies:

1. The teacher should let the student become familiar with a ruler before doing this activity.
2. Let the student use the ruler to help answer the questions.

ACTIVITY 4:

Suggested materials
worksheet

Directions to the student:


start
Read the fraction
Is there a number by which the numerator and denominator can be divided?
no
The fraction is in lowest terms. Write it.
stop
Reduce the fraction
yes
Divide both numerator and denominator by the number

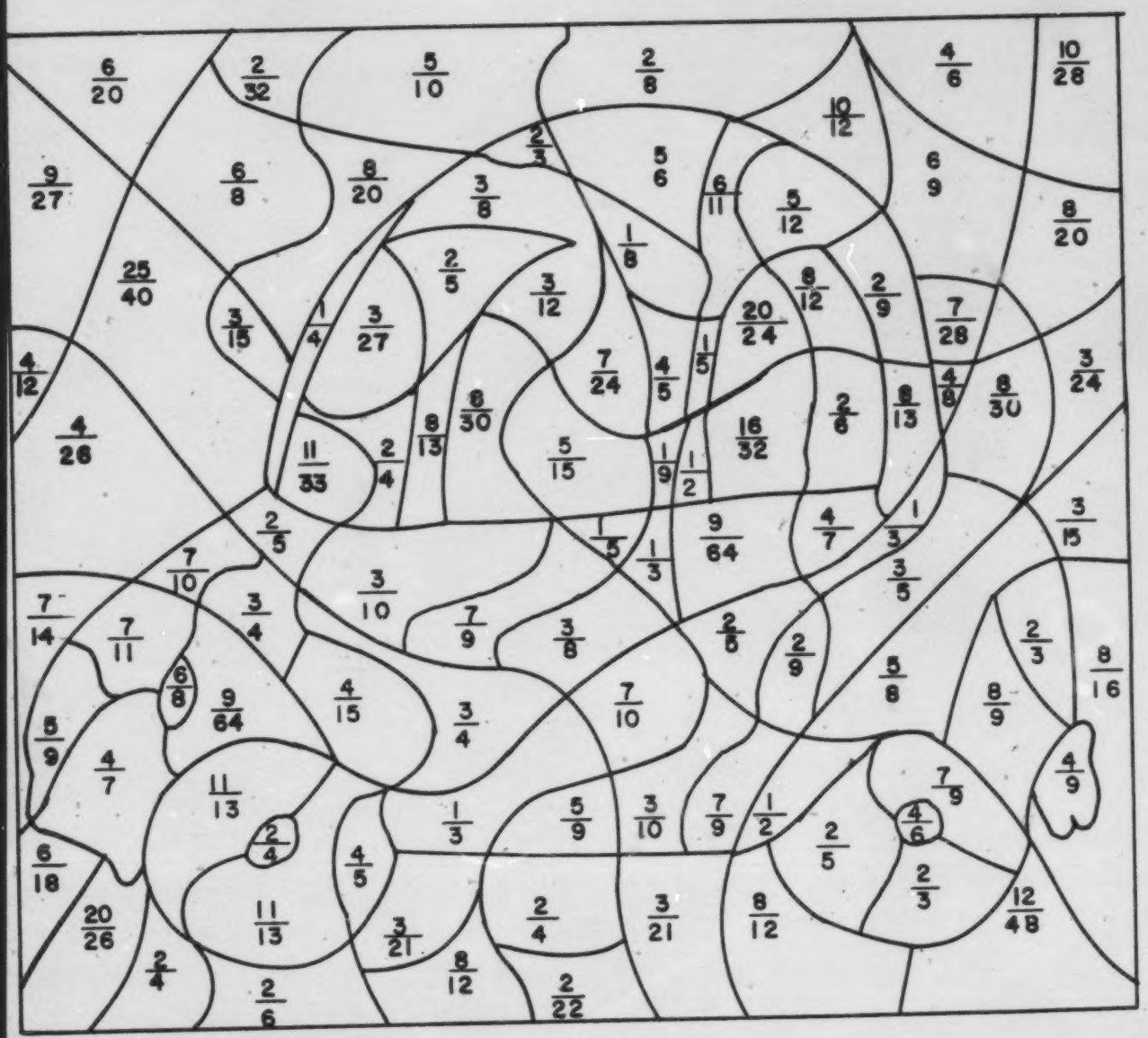
Use the flow chart to help you complete the table.

Fraction	Number by which both numerator and denominator must be divided	Fraction in lowest terms
6/8	2	3/4
4/8		
5/10		
12/16		
9/12		
6/9		
15/20		
8/10		
12/36		
10/20		
10/25		

Suggested strategies:
Students should be familiar with flow charting.

ACTIVITY 5:

Directions to Students:
Shade in the sections that have fractions in lowest terms.



ACTIVITY 6:

Suggested materials:

A set of 34 cards with one of the following fractions on each card.
10/20, 18/36, 4/12, 8/24, 4/6, 12/18, 4/8, 8/32, 9/12,

12/16, 5/25, 6/30, 10/25, 18/45, 6/15, 30/50, 16/20,

28/35, 3/18, 2/12, 25/30, 15/18, 2/16, 4/32, 9/24, 15/40,

15/24, 20/32, 31/27, 6/36, 10/18, 35/63, 35/45, and 14/18

One set for every five students.

Directions to the student:

Make a grid like the one below on your paper.

Choose any nine of the following fractions and put them in any box you wish, one in each of the nine boxes on your grid

1/2, 1/3, 2/9, 1/4, 3/4, 1/5, 2/5, 3/5, 4/5, 1/6, 5/6,

1/8, 3/8, 7/8, 1/9, 5/9, 7/9.

After everyone has completed filling in his grid, one person turns over one of the cards. Find the equivalent fraction in lowest terms to the fraction on the card and then check your grid. If you have the fraction in lowest terms on your grid, cross it out. If not, do nothing to your grid. Turn over the next card and repeat the procedure. The first one to get three in a row, across, down, or diagonally is the winner. When someone has three in a row everyone checks to be sure he is right. Make a new grid and play again.

Suggested strategies:

1. You can make this a class activity or play in small groups, say four or six students.

2. If the students are playing in groups, you may have to read the directions with them.
3. You can have several grids dittoed on one paper for the students to use.
4. Students may need help on how to fill in the grid. Show them an example.

APPENDIX B

Correspondence

THE UNIVERSITY OF NEBRASKA–LINCOLN
LINCOLN, NEBRASKA 68508

EACHERS COLLEGE
DEPARTMENT OF
ONDARY EDUCATION

July 27, 1971

Dear

I hope you will forgive this form letter, but it is the most expeditious means of conveying information. My committee has approved my proposal with some modifications and therefore, I am going to ask that you participate in an evaluation of the materials developed by the participants in the summer conference.

The enclosed information should give you a better understanding of what I am asking you to do. Also there is an information sheet that I would like to have completed and returned as soon as possible since it is important that I know how many units will be used.

Your willingness to help me in this evaluation is greatly appreciated. My present plans include visitations to some of the participating schools later this fall.

The mimeographed units have been mailed to all participants. If you have not received your units by August 14, please write, and another copy will be mailed.

Thank you again for offering to help with this study.

Sincerely yours,

Jack L. Beal

APPENDIX C

Directions for Evaluation of the Units

Information Sheet

Directions for Evaluation of the Units

1. Select one experimental and one control class.

2. The control class is to be taught using regular classroom methods and materials.

3. Where possible, the entire experimental class will be taught using the prepared units that you have selected rather than the regular textbook materials.

   For example, if you choose to use the unit on equivalent fractions found on pages 20-26, would you use the prepared materials rather than the textbook materials. When the unit materials are completed, then continue with the next topic in your textbook. It should not be any more difficult to use the unit materials than the regular textbook materials and this would strengthen the study. However, if it is impossible to use the units in place of the textbook materials, please use the units to supplement rather than to replace the regular textbook and continue to participate in the study.

4. A checklist will be completed after each unit is used. (This will be mailed to you later this summer.)

5. You will be asked to administer "The Revised Mathematics Attitude Test" at the beginning and again at the end of the semester. (This will be mailed to you later this summer and will take less than 15 minutes to administer.)

6. Assuming random assignment, either by a table of random numbers or by computer assignment, only a post-test will be administered at the end of the first semester. This test will be developed by me and will be mailed to you sometime during the fall semester.

7. Additional directions will be mailed to you later this summer.

Information Sheet

Name: _____

Home Telephone Number _____ Home Address _____

School Telephone Number _____ School Address _____

Approximate number of students in the experimental class _____

Approximate number of students in the control class _____

Are the students assigned by computer?
Yes _____
No _____

If the above question was answered no, can they be assigned by use of random numbers to two groups?
Yes _____
No _____

First Semester Begins _____

First Semester Ends _____

You as a member of the workshop wrote excellent units. It is hoped you will use as many as possible. Please make a tentative list of those units you plan to use next year. Please use the index provided with the mimeographed units.

Do you have any questions regarding the study? If yes, what are they?

APPENDIX D

Beal's Mathematics Competency Test

Directions:

Do not start until the examiner tells you to do so. You may answer questions even when you are not perfectly sure that your answers are correct, but you should avoid wild guessing.

Read each question carefully and decide which one of the answers is best. Notice the letter opposite your choice. Then, on a separate answer sheet, make a heavy black mark in the space under that letter.

For example:

Test booklet	Answer Sheet
5 + 8 = _____.	A. a b c d e

a. 11
b. 12
c. 13
d. 14
e. none of these

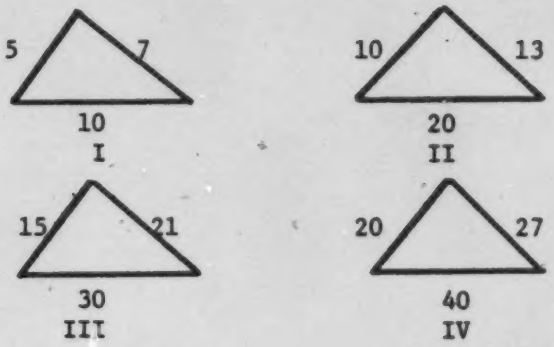
The correct answer is "13," which is answer c; so you would answer by making a heavy black mark that fills the space under the letter c. If the correct answer had not been given, you would have chosen answer e, "none of these."

When you finish a page, go on to the next page. Work as rapidly and accurately as you can.

When you are told to do so, open your test and begin work.

1. Which two of the four triangles are similar.

   a. I and II
   b. I and III
   c. I and IV
   d. II and III
   e. II and IV



2. 8 x 232 = _____

   a. 1846
   b. 1656
   c. 1646
   d. 1866
   e. none of these

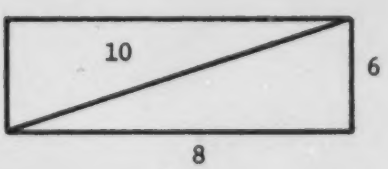
3. Which one of the numbers listed below makes the statement true?

   5 x ___ + 13 = 128

   a. 21
   b. 22
   c. 23
   d. 24
   e. none of these

4. The area of the following rectangle is _____ square units.

   a. 80
   b. 28
   c. 60
   d. 40
   e. none of these



5. When constructing the perpendicular bisector of line segment AB, we place the point of our compass on point A and mark an arc ____.

   a. above line segment AB.
   b. below line segment AB.
   c. above and below line segment AB
   d. through point B.
   e. none of these.

6. If 1 inch equals 2.54 centimeters, then 7 inches equals _____ centimeters.

   a. 0.36 2/7
   b. 177.8
   c. 17.78
   d. 3.6 2/7
   e. none of these

7. Following the pattern in the list below, choose the number that belongs in the blank.

   1, 4, 9, 16, 25, _____.

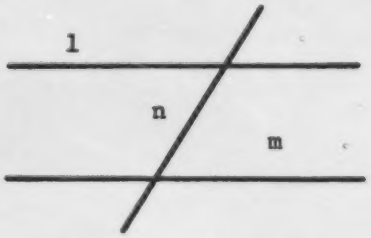
   a. 26
   b. 36
   c. 35
   d. 34
   e. none of these

8. If you have a jar containing 4 red marbles and 6 white marbles, the probability of drawing a red marble on any single draw is _____.

   a. 4/6
   b. 6/4
   c. 6/10
   d. 4/10
   e. none of these

9. Lines l and m lie in the same plane and may be best described as _____ lines.

   a. parallel
   b. perpendicular
   c. intersecting
   d. skew
   e. none of these



10. Suppose that you measure a board with a ruler divided into 1/8th inches and find that it is 8 inches long. What you mean is that it is _____.

    a. exactly 8 inches long.
    b. closer to 8 inches than to 8 1/8 inches long.
    c. closer to 8 1/8 inches than to 8 inches long.
    d. closer to 7 7/8 inches than to 8 inches long.
    e. none of these

11. Which one of the symbols < , > , = , and * will make the following statement true?

    3.52 _____ 3.5

    a. <
    b. >
    c. =
    d. *
    e. none of these

12. An angle has a measure of 5 degrees 34 minutes. The measure of the angle, rounded off to the nearest 10 minutes, is _____.

    a. 5 degrees 30 minutes
    b. 5 degrees 34 minutes
    c. 5 degrees 35 minutes
    d. 5 degrees 40 minutes
    e. none of these

13. 1/5 written as a decimal fraction is:

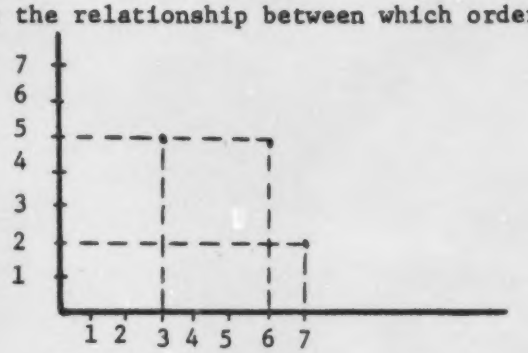
    a. 0.02
    b. 0.25
    c. 0.50
    d. 0.20
    e. none of these

14. Which one of the listed mathematical statements best describes the following verbal problem. If 7 is added to a number, the sum is 28. Find the number.

    a. n + 7 = 28
    b. n x 7 = 28
    c. 28 + 7 = n
    d. n + 28 = 7
    e. none of these

15. The graph below represents the relationship between which ordered pairs.

    a. (3,5) (5,6) (7,2)
    b. (5,3) (5,6) (7,2)
    c. (3,5) (6,5) (7,2)
    d. (2,7) (5,3) (6,5)
    e. none of these



16. John worked 48 hours one week. He received $2.84 per hour plus time and one-half for all hours over 40 hours. How much is John's earnings for the week?

a. $136.32
b. $ 52.50
c. $102.50
d. $113.60
e. none of these

17. 3/7 + 2/7 = _____

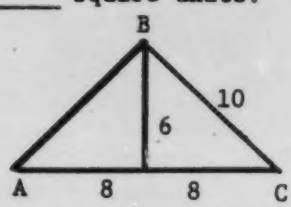
a. 6/7
b. 6/49
c. 5/14
d. 5/7
e. none of these

18. If a square is 1 foot on each side, then it has an area of 1 square foot or _____ square inches.

a. 144
b. 24
c. 12
d. 2
e. none of these

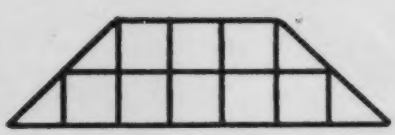
19. If the formula for the area of a triangle is 1/2 bh, then the area of triangle ABC is _____ square units.

a. 48
b. 36
c. 24
d. 96
e. none of these



20. What is the number of square units (area) in the region at the right?

a. 12
b. 11
c. 10
d. 9
e. none of these

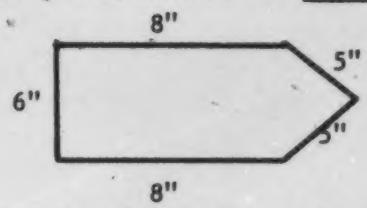


21. If there are 1352 students enrolled in a high school, then the number of enrolled students, rounded off to the nearest hundred, is _____.

a. 1352
b. 1360
c. 1000
d. 1400
e. none of these

22. The perimeter of the polygon pictured below is _____.

a. 60"
b. 38"
c. 42"
d. 32"
e. none of these



23. If a board is 2 feet 3 inches long, then it may be cut into _____ pieces each 1 inch long.

a. 27
b. 9
c. 15
d. 5
e. none of these

24. A board _____ long may be cut into exactly 5 boards each 2 feet 4 inches long.

a. 22 feet 8 inches
b. 10 feet 4 inches
c. 11 feet 8 inches
d. 3 feet 8 inches
e. none of these

25. Which of the listed mathematical statements best describes the following verbal problem. If 8 is subtracted from a number and the difference multiplied by 2, the result is 32. Find the number.

a. $n - 8 = 32$
b. $2(8-n) = 32$
c. $2(n-8) = 32$
d. $n - 8 = 32 \times 2$
e. none of these

26. 12% written as a common fraction is:

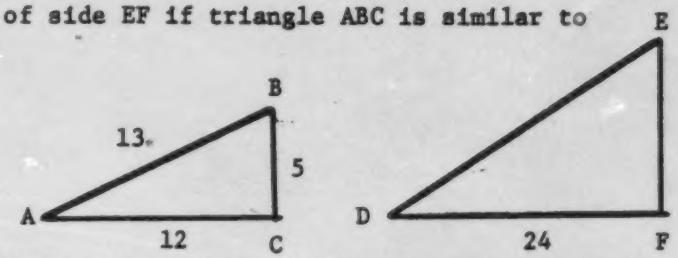
a. 1 1/5
b. 1/8
c. 3/25
d. 3/250
e. none of these

27. Which one of the following factions is <u>not</u> equivalent to 3/7?

a. 6/10
b. 21/49
c. 3/7
d. 9/21
e. 12/28

28. What is the length of side EF if triangle ABC is similar to triangle DEF?

a. 25
b. 2
c. 26
d. 10
e. none of these

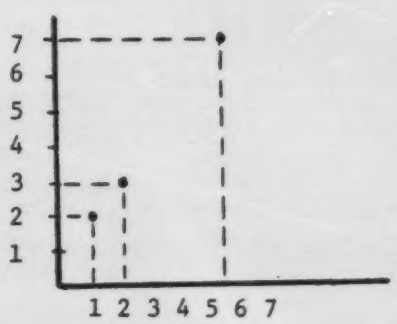


29. Assume your company allows you 15¢ per mile to drive your car plus $5 per day for meals and $10 per night for lodging. If you travel 350 miles in 5 days, your expense allowance is _____ (including 5 nights).

a. $127.50
b. $ 52.50
c. $102.50
d. $ 77.50
e. none of these

30. The graph below represents the relationship between which ordered pairs.

a. (1,2) (2,3) (5,7)
b. (2,1) (3,2) (7,5)
c. (1,2) (2,3) (5,6)
d. (2,1) (3,2) (6,5)

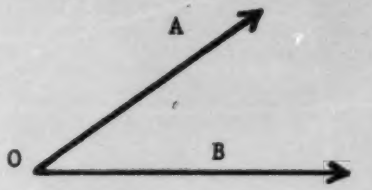


31. If two coins are flipped, the probability of obtaining 2 heads is:

    a. 1/2
    b. 1/4
    c. 1/8
    d. 3/4
    e. none of these

32. When constructing the angle bisector of $\angle AOB$, we place the point of our compass on O and draw an arc intersecting:

    a. only OA
    b. only OB
    c. OB and OA
    d. neither OA or OB
    e. none of these



33. If 2.2 pounds equals one kilogram, then 33 pounds equals _____ kilograms.

    a. 15
    b. 72.6
    c. 7.26
    d. 1.5
    e. none of these

34. Which of the following numbers make the statement true?

    $12 = 15 \times \_\_\_ - 48$

    a. 5
    b. 6
    c. 7
    d. 8
    e. none of these

35. Suppose you measure an angle with a protractor and find it to be 32 degrees. What you mean is that the measure of the angle is:

    a. exactly 32 degrees.
    b. closer to 32 degrees than to 31 degrees
    c. closer to 31 degrees than to 32 degrees.
    d. closer to 33 degrees than to 32 degrees.
    e. none of these

36. 5 2/3 - 1 1/5 = _____

a. 4 1/2
b. 4 13/15
c. 4 7/15
d. 6 13/15
e. none of these

37. Which one of the plane figures listed below does <u>not</u> have two pairs of parallel sides?

a. trapezoid
b. rectangle
c. rhombus
d. square
e. parallelogram

38. A rectangular prism has 6 sides that are rectangles. Besides it has _____.

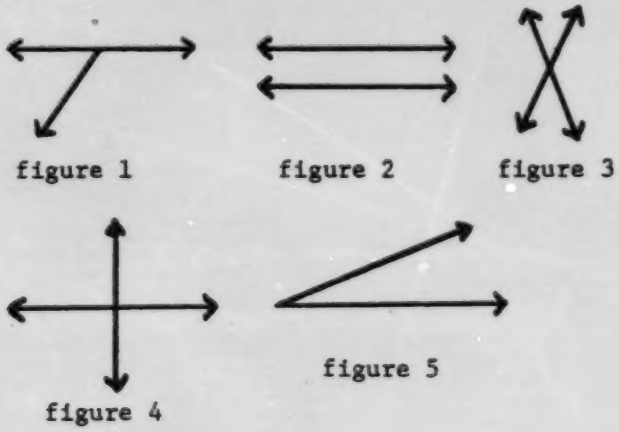
a. 6 edges and 6 corners
b. 12 edges and 8 corners
c. 6 edges and 8 corners
d. 8 edges and 6 corners
e. none of these

39. 13.02 - 5.96 = _____

a. 8.94
b. 7.06
c. 7.14
d. 8.06
e. none of these

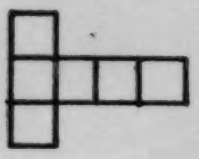
40. Which figure best shows two lines perpendicular to each other.

a. figure 1
b. figure 2
c. figure 3
d. figure 4
e. figure 5

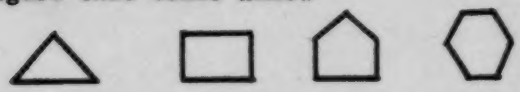


41. The pattern below may be folded into a _____.

a. cylinder
b. cube
c. sphere
d. pyramid
e. none of these



42. Following the pattern in the list below, choose the geometric figure that comes next.



a. figure 1
b. figure 2
c. figure 3
d. figure 4
e. none of these

figure 1

figure 2

figure 3

figure 4

43. If in measuring the length of 2 wires, we get one wire 1 centimeter and one wire 20 millimeters, then the total length of wire is _____ millimeters.

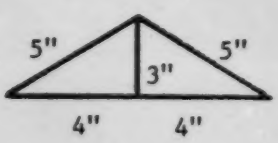
a. 21
b. 120
c. .120
d. 1.20
e. none of these

44. If you were to estimate the length of line segment AB, then you would say it is approximately _____ long.

a. 2 centimeters
b. 2 millimeters
c. 2 meters
d. 2 decimeters
e. none of these

45. The perimeter of the triangle pictured below is _____.

a. 12"
b. 18"
c. 48"
d. 24"
e. none of these

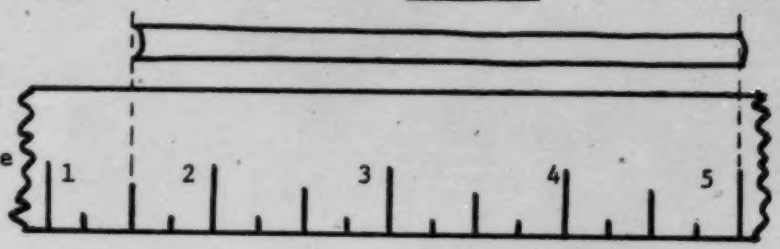


46. Which of the following fractions is equivalent to 3/13?

a. 9/19
b. 11/52
c. 11/21
d. 15/64
e. none of these

47. The length of the rod, as picutred below, is _____.

a. 5 inches
b. 1 1/2 inches
c. 3 1/2 inches
d. 3 inches
e. none of these



48. A triangle with 3 equal sides is said to be _____.

a. obtuse
b. scalene
c. isosceles
d. equilateral
e. none of these

49. How much money should you receive in change if you pay for a pizza that costs $3.63 with a 10 dollar bill?

a. 6 dollars 63 cents
b. 7 dollars 63 cents
c. 6 dollars 37 cents
d. 7 dollars 37 cents
e. none of these

50. 156.7 - 14.92 = _____

a. 8.94
b. 7.06
c. 7.14
d. 8.06
e. none of these

51. 950 ÷ 38 = _____

a. 24
b. 26
c. 36
d. 34
e. none of these

52. Which one of the symbols < , > , = , and * will make the following statement true?

7/12 _____ 28/48

a. <
b. >
c. =
d. *
e. none of these

APPENDIX E

Letter Requesting Permission to Use The Revised Math Attitude Scale

Reply

Copy of The Revised Math Attitude Scale

THE UNIVERSITY OF NEBRASKA-LINCOLN
LINCOLN, NEBRASKA 68508

ACHERS COLLEGE
DEPARTMENT OF
ONDARY EDUCATION

September 1, 1971

Dr. Lewis R. Aiken
Department of Psychology
University of Georgia
Athens, Georgia 30601

Dear Dr. Aiken:

I would like to reproduce your Revised Math Attitude Scale and use it in a study that I am conducting this fall. This study is being done as part of my work for my Ph.D. degree. With your permission, I will make plans to use it this fall.

Sincerely yours,

Jack L. Beal
Graduate Assistant
Secondary Education

JLB:pd

Permission granted. Please send summary of your findings when available.

Lewis Aiken

The Revised Math Attitude Scale

Directions: Please write your name in the upper right hand corner. Each of the statements on this opinionnaire expresses a feeling which a particular person has toward mathematics. You are to express, on a five-point scale, the extent of agreement between the feeling expressed in each statement and your own personal feeling. The five points are: Strongly Disagree (SD), Disagree (D), Undecided (U), Agree (A), and Strongly Agree (SA). You are to encircle the letter(s) which best indicates how closely you agree or disagree with the feeling expressed in each statement AS IT CONCERNS YOU.

1.	I am always under a terrible strain in a math class.	SD	D	U	A	SA
2.	I do not like mathematics, and it scares me to have to take it.	SD	D	U	A	SA
3.	Mathematics is very interesting to me, and I enjoy math courses.	SD	D	U	A	SA
4.	Mathematics is fascinating and fun.	SD	D	U	A	SA
5.	Mathematics makes me feel secure and at the same time it is stimulating.	SD	D	U	A	SA
6.	My mind goes blank, and I am unable to think clearly when working math.	SD	D	U	A	SA
7.	I feel a sense of insecurity when attempting mathematics.	SD	D	U	A	SA
8.	Mathematics makes me feel uncomfortable, restless, irritable, and impatient.	SD	D	U	A	SA
9.	The feeling that I have toward mathematics is a good feeling.	SD	D	U	A	SA
10.	Mathematics makes me feel as though I'm lost in a jungle of numbers and can't find my way out.	SD	D	U	A	SA
11.	Mathematics is something which I enjoy a great deal.	SD	D	U	A	SA
12.	When I hear the word math, I have a feeling of dislike.	SD	D	U	A	SA
13.	I approach math with a feeling of hesitation, resulting from a fear of not being able to do math.	SD	D	U	A	SA
14.	I really like mathematics.	SD	D	U	A	SA
15.	Mathematics is a course in school which I have always enjoyed studying.	SD	D	U	A	SA
16.	It makes me nervous to even think about having to do a math problem.	SD	D	U	A	SA
17.	I have never liked math, and it is my most dreaded subject.	SD	D	U	A	SA
18.	I am happier in a math class than in any other class.	SD	D	U	A	SA

19.	I feel at ease in mathematics, and I like it very much.	SD	D	U	A	SA
20.	I feel a definite positive reaction to mathematics; it's enjoyable.	SD	D	U	A	SA

APPENDIX F

Percentage Distribution of Responses by Competency
for Each Class Beal's Mathematics
Competency Test

CLASS 1 A

PERCENTAGE DISTRIBUTION OF RESPONSES BY COMPETENCY
BEAL'S MATHEMATICS COMPETENCY TEST

Competency	Test Item	Distribution of Responses a	b	c	d	e	Omit	Key
1	7	0	30	10	45	5	10	b
	42	15	20	10	20	0	35	b
2	13	10	0	10	20	40	20	d
	26	15	20	5	0	30	30	c
3	27	20	15	20	10	5	30	a
	46	20	0	15	10	25	30	e
4	2	5	10	5	0	70	10	e
	51	5	15	15	10	35	20	e
5	17	10	10	15	55	0	10	d
	36	25	15	10	10	15	25	c
6	39	15	0	20	20	20	30	b
	50	0	5	15	10	50	20	e

CLASS 1 B

PERCENTAGE DISTRIBUTION OF RESPONSES BY COMPETENCY
BEAL'S MATHEMATICS COMPETENCY TEST

Competency	Test Item	Distribution of Responses a	b	c	d	e	Omit	Key
1	7	0	24	12	41	6	17	b
	42	6	18	29	6	12	29	b
2	13	6	12	18	24	24	16	d
	26	6	12	6	6	18	52	c
3	27	59	0	0	6	6	29	a
	46	6	12	12	6	24	40	e
4	2	6	24	6	0	53	11	e
	51	6	6	6	6	53	23	e
5	17	6	6	18	41	6	23	d
	36	6	12	29	6	12	35	c
6	39	0	29	41	0	0	30	b
	50	0	6	12	0	47	35	e

CLASS 2 A

PERCENTAGE DISTRIBUTION OF RESPONSES BY COMPETENCY
BEAL'S MATHEMATICS COMPETENCY TEST

Competency	Test Item	Distribution of Responses a	b	c	d	e	Omit	Key
4	2	9	19	0	3	69	0	e
	51	6	22	9	3	60	0	e
7	14	78	0	3	3	16	0	a
	25	3	19	44	22	12	0	c
13	4	3	9	13	3	72	0	e
	19	41	9	34	3	13	0	a
16	38	34	28	19	3	16	0	b
	41	0	100	0	0	0	0	b
17	24	13	16	22	6	40	3	c
	49	9	6	56	22	7	0	c
20	10	31	38	13	9	9	0	b
	35	56	31	3	6	4	0	c
21	43	6	28	9	19	38	0	b
	44	69	3	9	0	19	0	a
22	47	13	3	69	6	9	0	c
	20	19	3	72	3	0	3	c
23	12	66	6	13	3	9	3	a
	21	0	16	3	72	9	0	d

CLASS 2 B

PERCENTAGE DISTRIBUTION OF RESPONSES BY COMPETENCY
BEAL'S MATHEMATICS COMPETENCY TEST

Competency	Test Item	Distribution of Responses a	b	c	d	e	Omit	Key
4	2	3	24	0	6	67	0	e
	51	9	6	12	3	70	0	e
7	14	72	9	3	6	10	0	a
	25	6	24	33	18	19	0	c
13	4	12	6	12	0	70	0	e
	19	15	9	45	12	9	0	a
16	38	42	18	6	6	28	0	b
	41	3	85	3	0	9	0	b
17	24	12	27	33	6	22	0	c
	49	3	3	85	3	6	0	c
20	10	36	21	18	9	16	0	b
	35	79	15	0	3	3	0	c
21	43	15	18	3	12	52	0	b
	44	48	21	9	0	22	0	a
22	47	21	6	55	3	15	0	c
	20	18	0	58	3	21	0	c
23	12	48	3	6	9	34	0	a
	21	15	18	0	48	19	0	d

CLASS 3 A

PERCENTAGE DISTRIBUTION OF RESPONSES BY COMPETENCY
BEAL'S MATHEMATICS COMPETENCY TEST

Competency	Test Item	Distribution of Responses a	b	c	d	e	Omit	Key
2	13	10	10	10	20	50	0	d
	26	10	20	10	0	50	10	c
3	27	50	10	20	10	10	0	a
	46	10	20	30	10	20	10	e
4	2	0	40	0	0	60	0	e
	51	10	20	10	0	60	0	e
5	17	0	0	20	80	0	0	d
	36	10	10	30	10	40	0	c
6	39	10	60	10	20	0	0	b
	50	0	10	10	20	60	0	e
9	15	20	30	20	20	10	0	c
	30	40	30	20	10	0	0	a
10	9	40	20	20	20	0	0	a
	40	0	30	30	30	10	0	d
11	37	40	20	20	10	10	0	a
	48	10	0	30	20	40	0	d
12	22	0	10	40	40	10	0	d
	45	30	40	0	0	30	0	b
13	4	0	20	0	0	80	0	e
	19	30	10	0	0	50	10	a
14	5	30	20	0	40	0	10	c
	32	0	10	30	30	20	10	c
15	1	20	20	0	30	30	0	b
	28	20	20	20	10	20	10	d
16	38	50	10	20	20	0	0	b
	41	0	60	10	10	20	0	b

CLASS 3 A (continued)

Competency	Test Item	Distribution of Responses a	b	c	d	e	Omit	Key
17	24	10	40	30	0	20	0	c
	49	10	20	60	10	0	0	c
18	18	0	10	70	10	10	0	a
	23	40	10	30	20	0	0	a
19	6	10	0	60	0	30	0	c
	33	10	40	10	30	10	0	a
21	43	40	30	20	10	0	0	b
	44	30	10	20	10	30	0	a
22	47	10	10	40	0	40	0	c
	20	50	10	10	10	10	10	c
23	12	50	20	30	0	0	0	a
	21	10	30	10	50	0	0	d
24	8	40	20	0	30	10	0	d
	31	60	10	0	10	20	0	b
25	16	50	20	0	20	10	0	e
	29	20	10	50	0	20	0	a

CLASS 3 B

PERCENTAGE DISTRIBUTION OF RESPONSES BY COMPETENCY
BEAL'S MATHEMATICS COMPETENCY TEST

Competency	Test Item	Distribution of Responses a	b	c	d	e	Omit	Key
2	13	7	21	14	14	44	0	d
	26	0	14	7	0	79	0	c
3	27	50	14	14	22	0	0	a
	46	21	14	14	22	29	0	e
4	2	14	29	7	7	43	0	e
	51	14	14	14	0	58	0	e
5	17	0	0	14	86	0	0	d
	36	21	7	22	0	50	0	c
6	39	0	57	21	7	15	0	b
	50	7	0	0	14	79	0	e
9	15	0	29	36	7	28	0	c
	30	71	14	7	8	0	0	a
10	9	57	14	29	0	0	0	a
	40	7	36	14	36	7	0	d
11	37	29	36	14	14	0	7	a
	48	21	21	14	14	30	0	d
12	22	7	0	7	79	7	0	d
	45	7	36	14	7	36	0	b
13	4	0	7	7	14	72	0	e
	19	7	29	29	7	28	0	a
14	5	21	14	29	21	15	0	c
	32	0	29	29	21	14	7	c
15	1	7	29	0	43	21	0	b
	28	7	0	29	36	28	0	d
16	38	29	21	14	14	22	0	b
	41	7	64	7	0	22	0	b

CLASS 3 B (continued)

Competency	Test Item	Distribution of Responses a	b	c	d	e	Omit	Key
17	24	7	36	36	7	14	0	c
	49	21	7	29	29	14	0	c
18	18	0	14	50	7	29	0	a
	23	57	14	14	7	8	0	a
19	6	10	14	57	14	15	0	c
	33	21	21	36	15	7	0	a
21	43	7	14	29	29	21	0	b
	44	57	7	29	7	0	0	a
22	47	14	21	50	7	8	0	c
	20	43	0	36	0	21	0	c
23	12	57	7	21	7	8	0	a
	21	0	14	7	71	8	0	d
24	8	50	14	7	14	15	0	d
	31	64	7	0	14	7	8	b
25	16	36	7	0	7	43	7	e
	29	14	14	15	15	42	0	a

CLASS 4 A

PERCENTAGE DISTRIBUTION OF RESPONSES BY COMPETENCY
BEAL'S MATHEMATICS COMPETENCY TEST

Competency	Test Item	Distribution of Responses a	b	c	d	e	Omit	Key
2	13	0	0	17	44	33	6	d
	26	5	17	39	0	39	0	c
3	27	94	0	0	6	0	0	a
	46	22	11	0	6	56	6	e
4	2	11	28	0	0	61	0	e
	51	0	0	6	0	83	11	e
5	17	0	6	11	83	0	0	d
	36	6	0	83	0	6	6	c
6	39	0	78	6	0	11	6	b
	50	0	0	6	0	90	6	e
9	15	11	28	39	11	11	11	c
	30	61	33	6	0	0	0	a
10	9	72	11	11	6	0	0	a
	40	6	22	17	28	22	6	d
11	37	44	6	39	0	6	6	a
	48	0	6	11	61	17	6	d
12	22	6	0	6	83	6	0	d
	45	11	39	6	6	33	6	b
13	4	6	6	0	11	78	0	e
	19	22	17	22	6	33	0	a
14	5	0	11	50	22	6	11	c
	32	6	6	56	11	17	6	c
15	1	11	44	0	6	39	0	b
	28	6	6	11	50	28	0	d
16	38	44	22	6	11	11	6	b
	41	6	83	0	0	6	6	b

CLASS 4 A (continued)

Competency	Test Item	Distribution of Responses a	b	c	d	e	Omit	Key
17	24	11	11	33	6	39	0	c
	49	6	0	78	6	6	6	c
18	18	39	17	28	0	17	0	a
	23	61	11	17	0	11	0	a
19	6	0	0	83	0	6	11	c
	33	22	28	11	17	11	11	a
21	43	22	11	6	11	44	6	b
	44	33	11	17	17	17	6	a
22	47	33	0	28	6	28	6	c
	20	28	0	39	11	22	0	c
23	12	83	6	0	0	11	0	a
	21	6	6	6	83	0	0	d
24	8	78	22	0	0	0	0	d
	31	72	0	6	0	17	6	b
25	16	28	0	6	0	67	0	e
	29	50	6	11	0	33	0	a

CLASS 4 B

PERCENTAGE DISTRIBUTION OF RESPONSES BY COMPETENCY
BEAL'S MATHEMATICS COMPETENCY TEST

Competency	Test Item	Distribution of Responses a	b	c	d	e	Omit	Key
2	13	0	0	23	54	23	0	d
	26	0	8	31	0	62	0	c
3	27	100	0	0	0	0	0	a
	46	8	8	8	23	54	0	e
4	2	8	15	8	8	62	0	e
	51	23	0	8	0	62	8	e
5	17	0	0	8	85	7	0	d
	36	0	0	46	15	39	0	c
6	39	8	85	0	8	0	0	b
	50	0	8	0	0	92	0	e
9	15	0	8	54	15	23	0	c
	30	54	31	8	8	0	0	a
10	9	85	15	0	0	0	0	a
	40	8	31	0	46	15	0	d
11	37	62	9	31	0	0	0	a
	48	8	0	31	54	8	0	d
12	22	8	0	15	62	15	0	d
	45	0	61	8	8	23	0	b
13	4	16	0	15	15	54	0	e
	19	46	8	15	8	23	0	a
14	5	8	8	77	8	0	0	c
	32	0	0	62	8	30	0	c
15	1	23	38	0	15	24	0	b
	28	46	0	15	15	23	0	d
16	38	46	38	8	0	8	0	b
	41	0	85	0	15	0	0	b

CLASS 4 B (continued)

Competency	Test Item	Distribution of Responses a	b	c	d	e	Omit	Key
17	24	0	8	54	23	15	0	c
	49	0	15	77	0	8	0	c
18	18	23	15	54	0	8	0	a
	23	77	8	0	8	7	0	a
19	6	0	0	85	0	15	0	c
	33	23	38	0	15	23	0	a
21	43	15	0	8	23	54	0	b
	44	31	23	8	15	23	0	a
22	47	15	8	62	0	15	0	c
	20	31	0	38	0	31	0	c
23	12	69	23	0	0	8	0	a
	21	0	0	8	85	7	0	d
24	8	69	8	0	15	8	0	d
	31	54	8	0	8	30	0	b
25	16	31	0	8	8	53	0	e
	29	46	0	15	15	23	0	a

CLASS 5 A

PERCENTAGE DISTRIBUTION OF RESPONSES BY COMPETENCY
BEAL'S MATHEMATICS COMPETENCY TEST

Competency	Test Item	Distribution of Responses a	b	c	d	e	Omit	Key
2	13	5	5	25	5	50	0	d
	26	5	15	25	5	50	0	c
3	27	80	5	5	0	10	0	a
	46	20	0	5	20	55	0	e
4	2	10	25	0	0	65	0	e
	51	10	10	10	0	65	5	e
5	17	10	0	15	75	0	0	d
	36	20	10	40	0	30	0	c
6	39	25	55	0	10	10	0	b
	50	5	5	0	5	80	5	e
9	15	20	20	20	10	30	0	c
	30	55	35	0	5	0	5	a
10	9	65	20	10	0	5	0	a
	40	5	45	10	25	15	0	d
11	37	25	0	55	5	15	0	a
	48	20	10	10	40	15	5	d
12	22	0	5	10	70	15	0	d
	45	10	45	10	25	10	0	b
13	4	5	10	0	5	80	0	e
	19	15	20	35	10	20	0	a
14	5	35	5	50	10	0	0	c
	32	20	10	35	25	10	0	c
15	1	30	35	5	15	15	0	b
	28	5	0	20	55	20	0	d
16	38	45	15	10	10	20	0	b
	41	5	65	0	10	15	5	b

CLASS 5 A (continued)

Competency	Test Item	Distribution of Responses a	b	c	d	e	Omit	Key
17	24	5	10	40	0	45	0	c
	49	10	0	65	15	10	0	c
18	18	20	0	55	5	20	0	a
	23	80	0	0	15	5	0	a
19	6	0	25	75	0	0	0	c
	33	20	35	15	5	15	10	a
21	43	20	20	0	20	40	0	b
	44	35	15	0	15	35	0	a
22	47	20	5	55	0	20	0	c
	20	20	0	50	15	15	0	c
23	12	65	0	20	5	10	0	a
	21	0	20	0	60	10	10	d
24	8	65	15	5	10	0	5	d
	31	50	25	10	5	10	0	b
25	16	50	25	10	5	10	0	b
	19	45	5	0	20	30	0	c

CLASS 5 B

PERCENTAGE DISTRIBUTION OF RESPONSES BY COMPETENCY
BEAL'S MATHEMATICS COMPETENCY TEST

Competency	Test Item	Distribution of Responses a	b	c	d	e	Omit	Key
2	13	7	14	7	43	29	0	d
	26	0	0	7	0	93	0	c
3	27	79	0	21	0	0	0	a
	46	14	0	0	14	64	8	e
4	2	0	14	0	0	86	0	e
	51	14	7	7	0	64	8	e
5	17	0	0	7	86	7	0	d
	36	0	7	50	7	36	0	c
6	39	7	29	14	21	29	0	b
	50	0	7	0	7	79	7	e
9	15	14	0	57	7	14	7	c
	30	79	7	14	0	0	0	a
10	9	64	29	0	7	0	0	a
	40	7	21	7	29	36	0	d
11	37	57	7	21	0	14	0	a
	48	0	7	21	36	29	7	d
12	22	0	0	0	93	7	0	d
	45	7	64	0	7	22	0	b
13	4	7	7	7	14	65	0	e
	19	29	14	21	7	29	0	a
14	5	14	14	21	43	7	0	c
	32	0	0	50	36	14	0	c
15	1	21	36	21	7	14	0	b
	28	7	14	14	57	7	0	d
16	38	36	29	0	21	14	0	b
	41	0	93	0	0	7	0	b

CLASS 5 B (continued)

Competency	Test Item	Distribution of Responses a	b	c	d	e	Omit	Key
17	24	7	7	50	14	22	0	c
	49	0	0	93	0	0	7	c
18	18	14	7	50	7	14	8	a
	23	50	7	7	14	22	0	a
19	6	0	0	86	0	14	0	c
	33	29	21	14	14	22	0	a
21	43	29	21	14	0	36	0	b
	44	64	7	21	8	0	0	a
22	47	7	0	50	0	36	7	c
	20	29	9	57	0	14	0	c
23	12	64	21	7	8	0	0	a
	21	0	0	7	86	7	0	d
24	8	64	0	0	36	0	0	d
	31	93	0	0	0	7	0	b
25	16	36	0	7	7	50	0	e
	29	29	21	7	7	29	7	a

CLASS 6 A

PERCENTAGE DISTRIBUTION OF RESPONSES BY COMPETENCY
BEAL'S MATHEMATICS COMPETENCY TEST

Competency	Test Item	Distribution of Responses						Key
		a	b	c	d	e	Omit	
1	7	4	48	18	15	15	0	b
	42	22	22	19	7	19	11	b
2	13	7	22	15	0	52	4	d
	26	7	30	7	0	48	8	c
3	27	48	22	7	4	19	0	a
	46	11	0	33	15	30	11	e
4	2	0	22	0	4	74	0	e
	51	15	11	4	15	41	14	e
5	17	18	19	0	56	4	3	d
	36	7	37	30	7	8	11	c
6	39	11	19	19	19	18	14	b
	50	26	11	11	11	30	11	e

CLASS 6 B

PERCENTAGE DISTRIBUTION OF RESPONSES BY COMPETENCY
BEAL'S MATHEMATICS COMPETENCY TEST

Competency	Test Item	Distribution of Responses a	b	c	d	e	Omit	Key
1	7	0	15	8	38	19	0	b
	42	8	23	19	31	12	7	b
2	13	12	8	15	8	54	3	d
	26	15	23	19	12	27	4	c
3	27	42	12	19	12	15	0	a
	46	19	31	8	8	27	7	e
4	2	7	27	8	4	54	0	e
	51	12	23	23	8	19	15	e
5	17	23	12	19	27	12	7	d
	36	19	15	23	8	19	16	c
6	39	19	27	27	8	19	0	b
	50	12	12	35	15	16	0	e

CLASS 7 A

PERCENTAGE DISTRIBUTION OF RESPONSES BY COMPETENCY
BEAL'S MATHEMATICS COMPETENCY TEST

Competency	Test Item	Distribution of Responses a	b	c	d	e	Omit	Key
1	7	0	50	0	28	22	0	b
	42	6	56	16	17	5	0	b
2	13	0	0	17	78	5	0	d
	26	0	17	50	0	33	0	c
3	27	94	0	0	6	0	0	a
	46	0	0	0	0	100	0	e
4	2	0	17	0	0	83	0	e
	51	0	0	6	0	94	0	e
10	9	78	17	5	0	0	0	a
	40	17	39	0	44	0	0	d
11	37	44	0	39	0	17	0	a
	48	6	33	6	50	5	0	d

CLASS 7 B

PERCENTAGE DISTRIBUTION OF RESPONSES BY COMPETENCY
BEAL'S MATHEMATICS COMPETENCY TEST

Competency	Test Item	Distribution of Responses a	b	c	d	e	Omit	Key
1	7	0	28	5	28	39	0	b
	42	6	50	6	16	11	11	b
2	13	5	0	6	72	17	0	d
	26	5	11	28	0	56	0	c
3	27	94	0	0	0	0	6	a
	46	6	0	0	11	78	5	e
4	2	0	11	0	0	89	0	e
	51	0	5	6	0	83	6	e
10	9	72	17	0	0	11	0	a
	40	5	33	0	56	0	6	d
11	37	6	0	78	0	16	0	a
	48	0	0	22	67	5	6	d

CLASS 8 A

PERCENTAGE DISTRIBUTION OF RESPONSES BY COMPETENCY
BEAL'S MATHEMATICS COMPETENCY TEST

Competency	Test Item	Distribution of Responses a	b	c	d	e	Omit	Key
1	7	67	0	16	0	0	17	b
	42	17	83	0	0	0	0	b
2	13	0	0	100	0	0	0	d
	26	0	0	33	0	67	0	c
3	27	100	0	0	0	0	0	a
	46	0	0	0	0	100	0	e
4	2	0	67	0	0	33	0	e
	51	0	0	0	100	0	0	e
5	17	0	0	0	83	17	0	d
	36	0	0	100	0	0	0	c
6	39	0	50	0	33	17	0	b
	50	0	0	0	100	0	0	e
7	14	100	0	0	0	0	0	a
	25	0	33	67	0	0	0	c
8	3	0	0	100	0	0	0	c
	34	0	0	0	0	100	0	e
9	15	67	0	16	17	0	0	c
	30	50	50	0	0	0	0	a
10	9	100	0	0	0	0	0	a
	40	0	33	0	67	0	0	d
11	37	34	16	33	0	17	0	a
	48	0	0	0	50	33	17	d
12	22	17	0	0	83	0	0	d
	45	0	50	17	17	16	0	d
13	4	0	33	0	0	67	0	e
	19	33	17	33	0	17	0	a

CLASS 8 A (continued)

Competency	Test Item	Distribution of Responses a	b	c	d	e	Omit	Key
14	5	0	0	16	67	17	0	c
	32	17	0	33	0	50	0	c
16	38	50	0	0	17	33	0	b
	41	0	67	0	0	33	0	b
17	24	0	50	33	0	17	0	c
	49	0	0	83	17	0	0	c

CLASS 8 B

PERCENTAGE DISTRIBUTION OF RESPONSES BY COMPETENCY
BEAL'S MATHEMATICS COMPETENCY TEST

Competency	Test Item	Distribution of Responses a	b	c	d	e	Omit	Key
1	7	67	0	16	0	0	17	b
	42	17	83	0	0	0	0	b
2	13	0	0	100	0	0	0	d
	26	0	0	33	0	67	0	c
3	27	100	0	0	0	0	0	a
	46	0	0	0	0	100	0	e
4	2	0	67	0	0	33	0	e
	51	0	0	0	100	0	0	e
5	17	0	0	0	83	17	0	d
	36	0	0	100	0	0	0	c
6	39	0	50	0	33	17	0	b
	50	0	0	0	100	0	0	e
7	14	100	0	0	0	0	0	a
	25	0	33	67	0	0	0	c
8	3	0	0	100	0	0	0	c
	34	0	0	0	0	100	0	e
9	15	67	0	16	17	0	0	c
	30	50	50	0	0	0	0	a
10	9	100	0	0	0	0	0	a
	40	0	33	0	67	0	0	d
11	37	34	16	33	0	17	0	a
	48	0	0	0	50	33	17	d
12	22	17	0	0	83	0	0	d
	45	0	50	17	17	16	0	d
13	4	0	33	0	0	67	0	e
	19	33	17	33	0	17	0	a

CLASS 8 B (continued)

Competency	Test Item	Distribution of Responses a	b	c	d	e	Omit	Key
14	5	6	11	33	39	11	0	c
	32	6	0	50	0	44	0	c
16	38	11	33	17	0	39	0	b
	41	0	72	0	6	22	0	b
17	24	0	11	44	6	33	6	c
	49	6	0	94	0	0	0	c

CLASS 9 A

PERCENTAGE DISTRIBUTION OF RESPONSES BY COMPETENCY
BEAL'S MATHEMATICS COMPETENCY TEST

Competency	Test Item	Distribution of Responses a	b	c	d	e	Omit	Key
1	7	7	36	7	21	21	8	b
	42	7	36	21	7	7	21	b
2	13	7	0	0	14	71	8	d
	26	0	21	7	0	64	7	c
4	2	0	14	0	0	86	0	d
	51	0	21	8	14	42	15	d
5	17	7	0	21	57	7	8	d
	36	14	7	29	14	36	0	c
18	18	14	21	36	14	7	8	a
	23	29	7	7	29	21	7	a
20	10	36	14	7	7	14	22	b
	35	50	14	7	7	14	8	b

CLASS 9 B

PERCENTAGE DISTRIBUTION OF RESPONSES BY COMPETENCY
BEAL'S MATHEMATICS COMPETENCY TEST

Competency	Test Item	Distribution of Responses a	b	c	d	e	Omit	Key
1	7	0	27	27	18	28	0	b
	42	0	36	9	27	0	28	
2	13	0	27	9	9	55	0	d
	26	18	0	9	0	54	19	c
4	2	0	18	0	0	82	0	d
	51	9	0	9	0	64	18	d
5	17	9	0	36	55	0	0	d
	36	27	18	18	9	18	10	c
18	18	0	9	45	36	10	0	a
	23	27	0	18	27	18	10	a
20	10	36	27	18	0	9	10	b
	35	73	18	0	0	0	9	b

CLASS 10 A

PERCENTAGE DISTRIBUTION OF RESPONSES BY COMPETENCY
BEAL'S MATHEMATICS COMPETENCY TEST

Competency	Test Item	Distribution of Responses a	b	c	d	e	Omit	Key
2	13	4	13	22	13	48	0	d
	26	4	13	17	4	62	0	c
3	27	70	13	4	8	0	5	a
	46	13	0	4	17	66	0	e
4	2	8	17	0	0	75	0	e
	51	13	9	22	9	47	0	e
5	17	4	4	4	83	5	0	d
	36	13	13	39	13	22	0	c
6	39	13	57	9	17	4	0	b
	50	0	9	4	0	87	0	e
10	9	83	14	0	3	0	0	a
	40	35	17	9	26	13	0	d
11	37	30	4	57	0	9	0	a
	48	4	13	35	35	13	0	d
12	22	0	0	0	65	35	0	d
	45	4	26	13	27	30	0	b
13	4	4	4	9	0	83	0	e
	19	17	13	13	13	44	0	a
14	5	39	9	4	35	13	0	c
	32	9	4	26	26	35	0	c
15	1	4	91	5	0	0	0	b
	28	9	4	30	43	14	0	d
17	24	9	13	43	9	26	0	c
	49	0	4	61	17	13	5	c
18	18	17	13	26	17	27	0	a
	23	61	0	4	13	22	0	a
19	6	9	0	65	13	9	4	c
	33	26	9	17	18	30	0	a

CLASS 10 B

PERCENTAGE DISTRIBUTION OF RESPONSES BY COMPETENCY
BEAL'S MATHEMATICS COMPETENCY TEST

Competency	Test Item	Distribution of Responses a	b	c	d	e	Omit	Key
2	13	6	0	13	71	10	0	d
	26	0	3	35	3	59	0	c
3	27	97	0	0	0	3	0	a
	46	13	3	0	6	78	0	e
4	2	0	29	0	3	68	0	e
	51	6	3	0	3	88	0	e
5	17	3	0	0	90	7	0	d
	36	3	3	94	0	0	0	c
6	39	0	87	0	6	7	0	b
	50	0	3	0	3	94	0	e
10	9	84	3	6	3	4	0	a
	40	6	6	3	81	4	0	d
11	37	39	19	39	0	3	0	a
	48	0	0	16	84	0	0	d
12	22	3	0	3	87	7	0	d
	45	6	65	6	0	23	0	b
13	4	3	13	0	0	84	0	e
	19	39	10	13	10	28	0	a
14	5	19	0	71	0	10	0	c
	32	0	0	52	10	38	0	c
15	1	26	48	6	10	3	7	b
	28	3	3	6	52	36	0	d
17	24	13	6	32	0	45	4	c
	49	0	0	87	13	0	0	c
18	18	26	23	19	10	22	0	a
	23	74	3	13	3	7	0	a
19	6	0	16	74	3	7	0	c
	33	19	35	13	16	13	4	a

CLASS 11 A

PERCENTAGE DISTRIBUTION OF RESPONSES BY COMPETENCY
BEAL'S MATHEMATICS COMPETENCY TEST

Competency	Test Item	Distribution of Responses a	b	c	d	e	Omit	Key
1	7	0	11	16	53	20	0	b
	42	21	5	21	21	5	27	b
2	13	5	0	21	32	42	0	d
	26	5	21	11	0	58	5	c
3	27	47	5	11	26	5	6	a
	46	26	0	11	5	32	26	e
4	2	0	16	5	0	79	0	e
	51	5	5	5	0	37	48	e
5	17	16	0	11	68	5	0	d
	36	16	5	42	5	21	11	c
6	39	5	47	5	11	16	16	b
	50	5	0	0	5	53	37	e

CLASS 11 B

PERCENTAGE DISTRIBUTION OF RESPONSES BY COMPETENCY
BEAL'S MATHEMATICS COMPETENCY TEST

Competency	Test Item	Distribution of Responses a	b	c	d	e	Omit	Key
1	7	0	43	0	29	28	0	b
	42	29	0	0	29	14	28	b
2	13	0	14	43	0	43	0	d
	26	14	14	0	0	72	0	c
3	27	57	14	14	0	0	15	a
	46	29	0	14	14	14	29	e
4	2	0	0	0	14	86	0	e
	51	14	0	0	14	57	15	e
5	17	43	0	0	57	0	0	d
	36	0	14	43	0	43	0	c
6	39	0	43	29	0	0	28	b
	50	0	0	0	0	86	14	e

CLASS 12 A

PERCENTAGE DISTRIBUTION OF RESPONSES BY COMPETENCY
BEAL'S MATHEMATICS COMPETENCY TEST

Competency	Test Item	Distribution of Responses a	b	c	d	e	Omit	Key
1	7	0	67	7	20	7	0	b
	42	0	40	13	13	14	20	b
2	13	0	7	7	80	7	0	d
	26	20	20	20	0	40	0	c
3	27	87	7	0	7	0	0	a
	46	20	0	7	0	33	40	e
4	2	0	40	0	0	60	0	e
	51	7	0	0	7	33	53	e
5	17	0	7	13	80	0	0	d
	36	27	13	40	0	20	0	c
6	39	7	47	13	20	7	6	b
	50	0	0	0	0	47	53	e

CLASS 12 B

PERCENTAGE DISTRIBUTION OF RESPONSES BY COMPETENCY
BEAL'S MATHEMATICS COMPETENCY TEST

Competency	Test Item	Distribution of Responses a	b	c	d	e	Omit	Key
1	7	0	50	14	21	7	8	b
	42	0	36	14	0	0	50	b
2	13	7	14	0	43	36	0	d
	26	14	29	29	0	28	0	c
3	27	93	7	0	0	0	0	a
	46	0	0	7	7	36	50	e
4	2	7	21	14	0	58	0	e
	51	0	0	0	7	14	79	e
5	17	14	7	7	72	0	0	d
	36	7	0	43	22	14	14	c
6	39	21	29	0	14	0	36	b
	50	0	0	0	7	36	57	e

CLASS 13 A

PERCENTAGE DISTRIBUTION OF RESPONSES BY COMPETENCY
BEAL'S MATHEMATICS COMPETENCY TEST

Competency	Test Item	Distribution of Responses a	b	c	d	e	Omit	Key
1	7	8	31	23	15	15	8	b
	42	0	39	15	15	0	31	b
2	13	0	23	15	31	31	0	d
	26	8	8	23	8	46	7	c
3	27	62	23	8	0	0	7	a
	46	38	0	0	23	8	31	e
4	2	0	38	0	0	62	0	e
	51	8	15	0	15	16	46	e
5	17	8	8	23	54	7	0	d
	36	31	8	15	8	31	7	c
6	39	23	23	23	15	0	16	b
	50	15	0	15	8	23	39	e

CLASS 13 B

PERCENTAGE DISTRIBUTION OF RESPONSES BY COMPETENCY
BEAL'S MATHEMATICS COMPETENCY TEST

Competency	Test Item	Distribution of Responses a	b	c	d	e	Omit	Key
1	7	0	64	7	14	15	0	b
	42	7	29	14	14	0	36	b
2	13	0	36	7	0	57	0	d
	26	0	14	21	7	58	0	c
3	27	50	14	21	7	0	8	a
	46	14	0	0	29	7	50	e
4	2	7	7	0	7	79	0	e
	51	0	21	0	0	14	65	e
5	17	7	0	21	72	0	0	d
	36	14	14	14	21	8	29	c
6	39	14	29	7	7	14	29	b
	50	0	14	7	0	29	50	e

CLASS 14 A

PERCENTAGE DISTRIBUTION OF RESPONSES BY COMPETENCY
BEAL'S MATHEMATICS COMPETENCY TEST

Competency	Test Item	Distribution of Responses a	b	c	d	e	Omit	Key
1	7	0	33	8	50	9	0	b
	42	8	25	17	17	8	25	b
2	13	8	8	25	9	50	0	d
	26	0	17	25	0	42	16	c
3	27	58	8	8	0	9	17	a
	46	17	0	8	33	17	25	e
4	2	8	33	0	0	59	0	e
	51	17	8	17	0	33	25	e
5	17	0	8	25	58	0	9	d
	36	17	17	17	17	16	16	c
6	39	8	42	8	0	17	25	b
	50	8	0	8	8	42	34	e

CLASS 14 B

PERCENTAGE DISTRIBUTION OF RESPONSES BY COMPETENCY
BEAL'S MATHEMATICS COMPETENCY TEST

Competency	Test Item	Distribution of Responses a	b	c	d	e	Omit	Key
1	7	8	33	8	17	34	0	b
	42	0	33	25	8	0	34	b
2	13	0	17	8	8	58	9	d
	26	8	17	8	0	58	9	c
3	27	67	8	8	8	0	9	a
	46	25	0	8	8	25	9	e
4	2	8	25	0	0	67	0	e
	51	0	8	17	8	25	42	e
5	17	0	0	25	58	8	9	d
	36	25	0	25	17	8	25	c
6	39	17	33	0	8	17	25	b
	50	0	17	0	0	42	41	e

CLASS 15 A

PERCENTAGE DISTRIBUTION OF RESPONSES BY COMPETENCY
BEAL'S MATHEMATICS COMPETENCY TEST

Competency	Test Item	Distribution of Responses a	b	c	d	e	Omit	Key
1	7	0	54	0	15	23	8	b
	42	31	15	8	23	8	15	b
2	13	0	15	8	62	15	0	d
	26	31	8	23	0	23	15	c
3	27	85	8	0	7	0	0	a
	46	0	8	8	8	53	23	e
4	2	0	31	0	8	61	0	e
	51	15	15	0	0	31	39	e
5	17	8	0	8	77	0	7	d
	36	8	0	54	0	23	15	c
6	39	0	38	23	0	23	16	b
	50	0	8	0	8	46	38	e

CLASS 15 B

PERCENTAGE DISTRIBUTION OF RESPONSES BY COMPETENCY
BEAL'S MATHEMATICS COMPETENCY TEST

Competency	Test Item	Distribution of Responses a	b	c	d	e	Omit	Key
1	7	0	50	0	17	33	0	b
	42	8	33	25	17	0	17	b
2	13	0	17	0	58	25	0	d
	26	0	17	42	0	33	8	c
3	27	92	0	8	0	0	0	a
	46	17	0	8	25	33	17	e
4	2	8	8	0	9	75	0	e
	51	0	0	17	0	58	25	e
5	17	8	0	17	75	0	0	d
	36	8	0	67	0	17	8	c
6	39	8	58	0	0	17	17	b
	50	0	0	0	0	83	17	e

APPENDIX G

Unit Summary

Unit Summary

The following questionnaire is to be completed after teaching each unit of the conference materials. Please return with test materials at the end of the semester.

1. Unit number _____ Objectives _____

2. Number of students using conference materials _____

3. Number of class periods devoted to the unit _____

4. Was the textbook used in conjunction with the conference materials?

   Yes _____ No _____

   If yes, approximately what percent of class time was devoted to conference materials?

5. Describe any additional materials used by the control class (other than the textbook) that might affect the final results of our testing.

6. Please list any suggestions you might have for improving the units just completed.